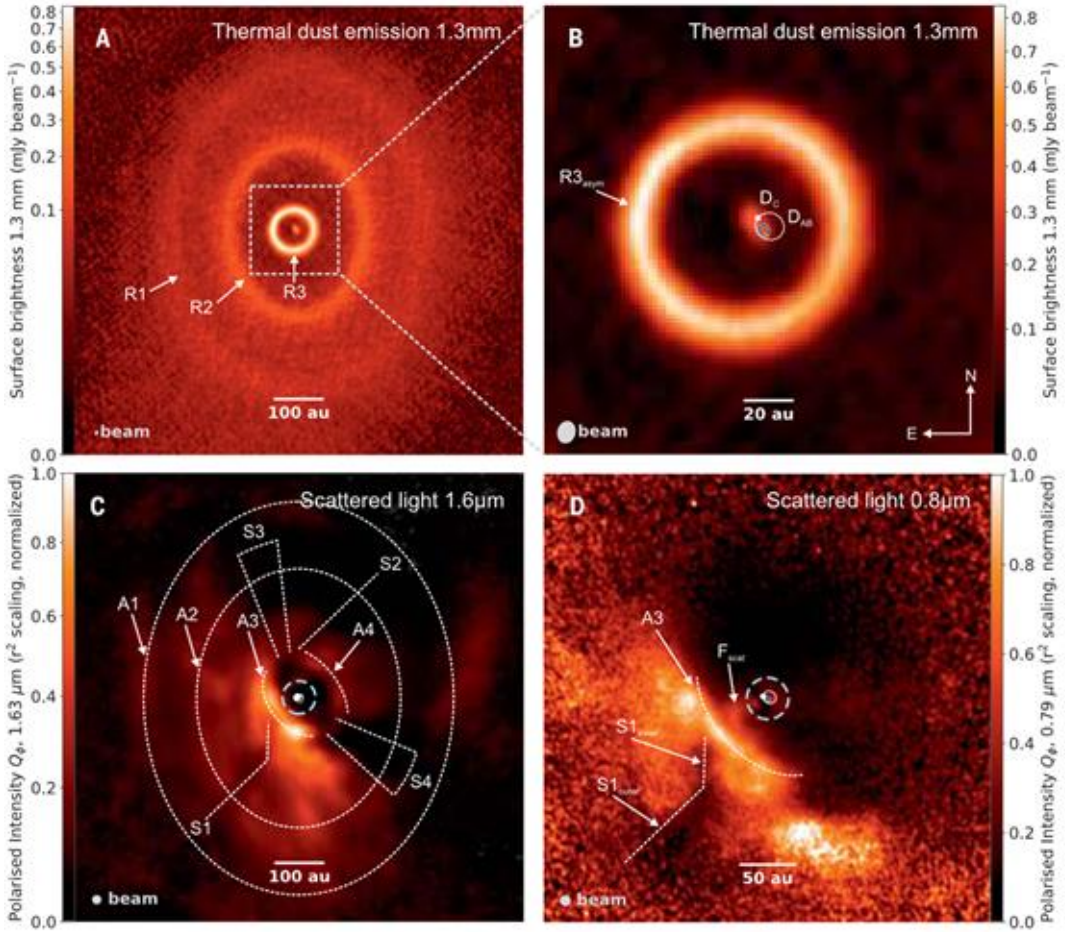
Steady Warps: Nonlinear Effects and Breaking

Jiaru Li - CIERA Postdoctoral Fellow, Northwestern University

in collaboration with Prof. Yoram Lithwick

June 24th, 2026 @ UChicago

Observational Signatures of Warped Protoplanetary Disks





Disk around GW Orionis
Kraus et al (2020)

Questions

Q1: How do disks bend and sustain coherent warps?

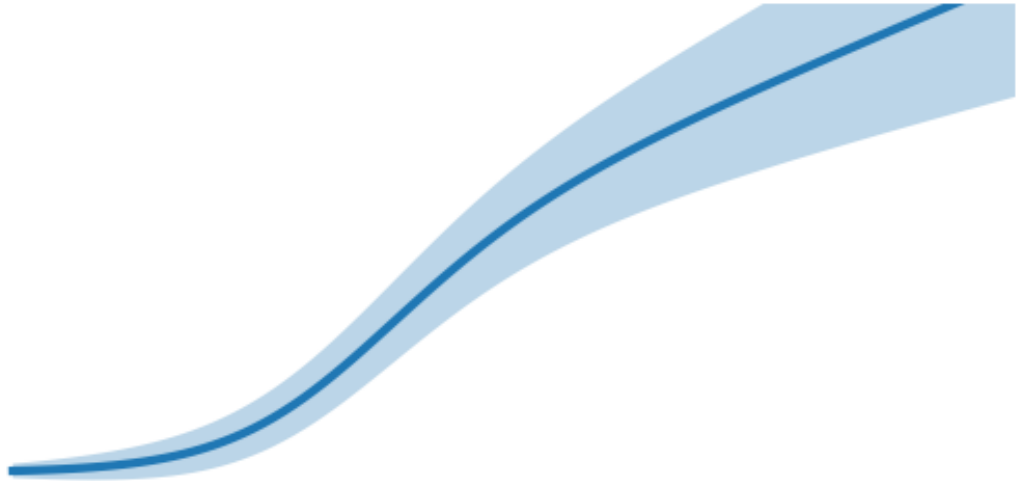
Q2: Under what conditions do they break into misaligned components?



Insights from the theory:

- Viscous parameter α → damping rate $\sim \alpha\Omega$..
- free warp: flatten over time..
- forced disk: settle into a (quasi-) steady state if the forcing is slow..

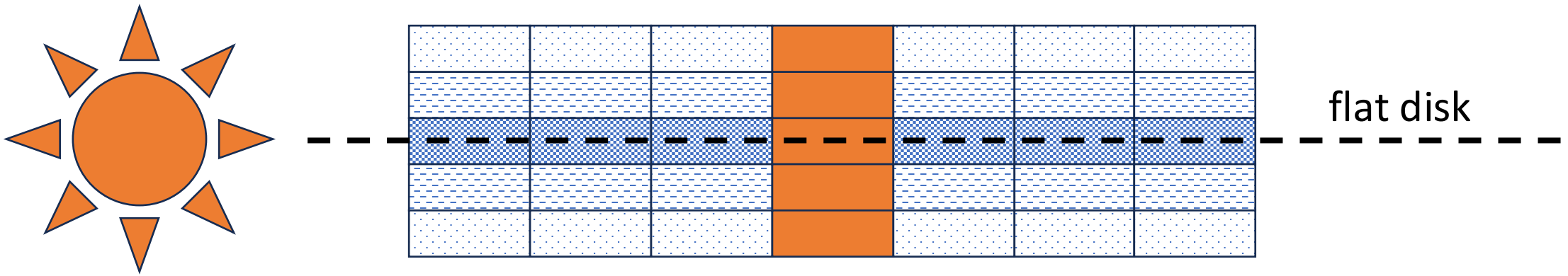




→ **Internal torque** **(this talk)**

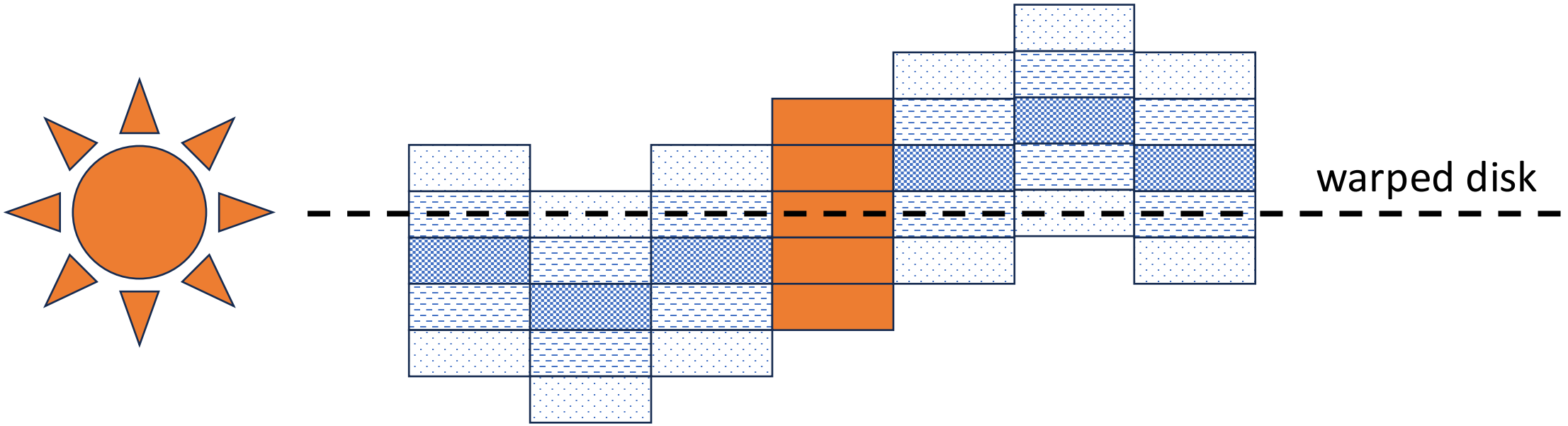
See also Papaloizou & Lin (1995), Ogilvie (1999), Lubow & Ogilvie (2000), Ogilvie & Latter (2013), Foucart & Lai (2014), Dullemond+ (2022), etc., for more about theories
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Warps 101: Flat Disk vs Warped Disk



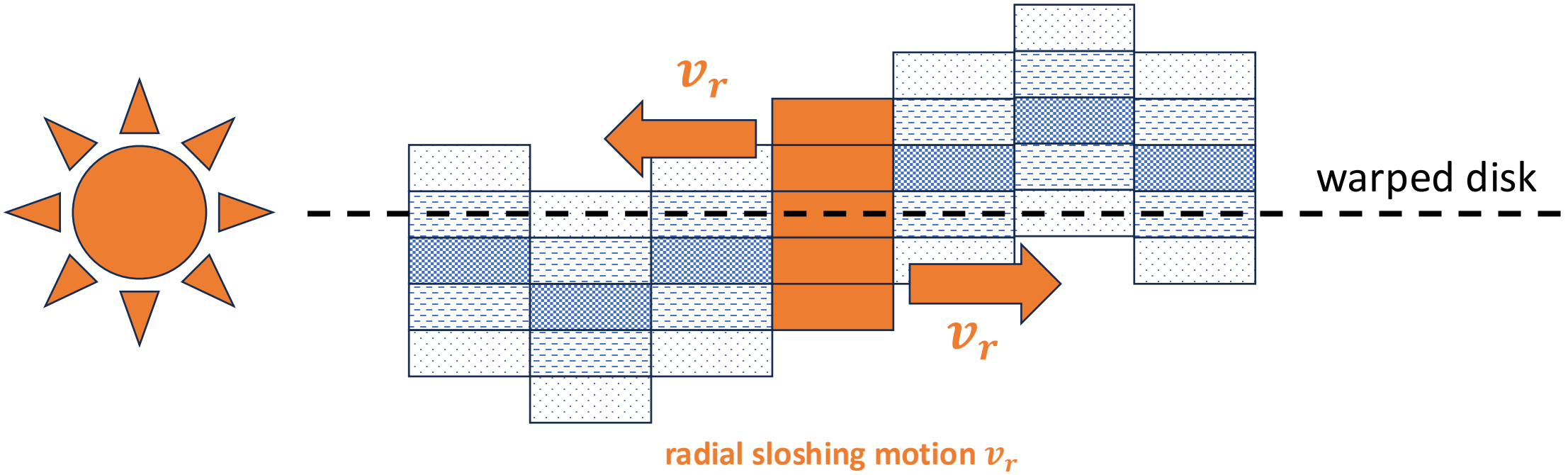
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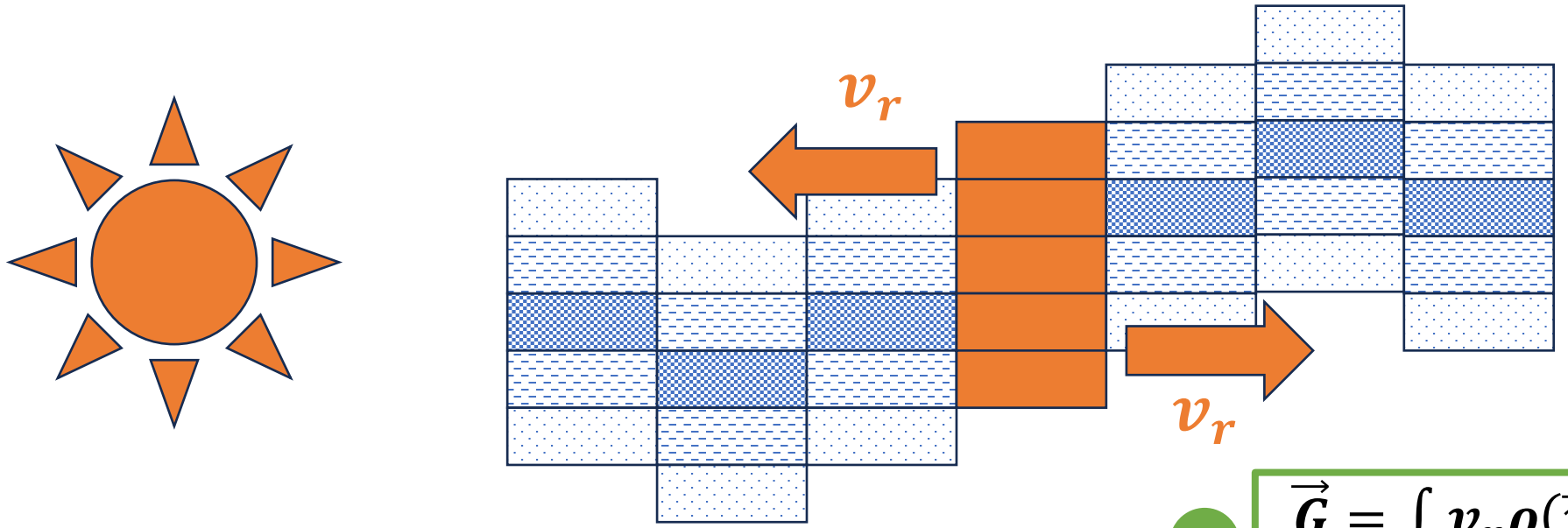
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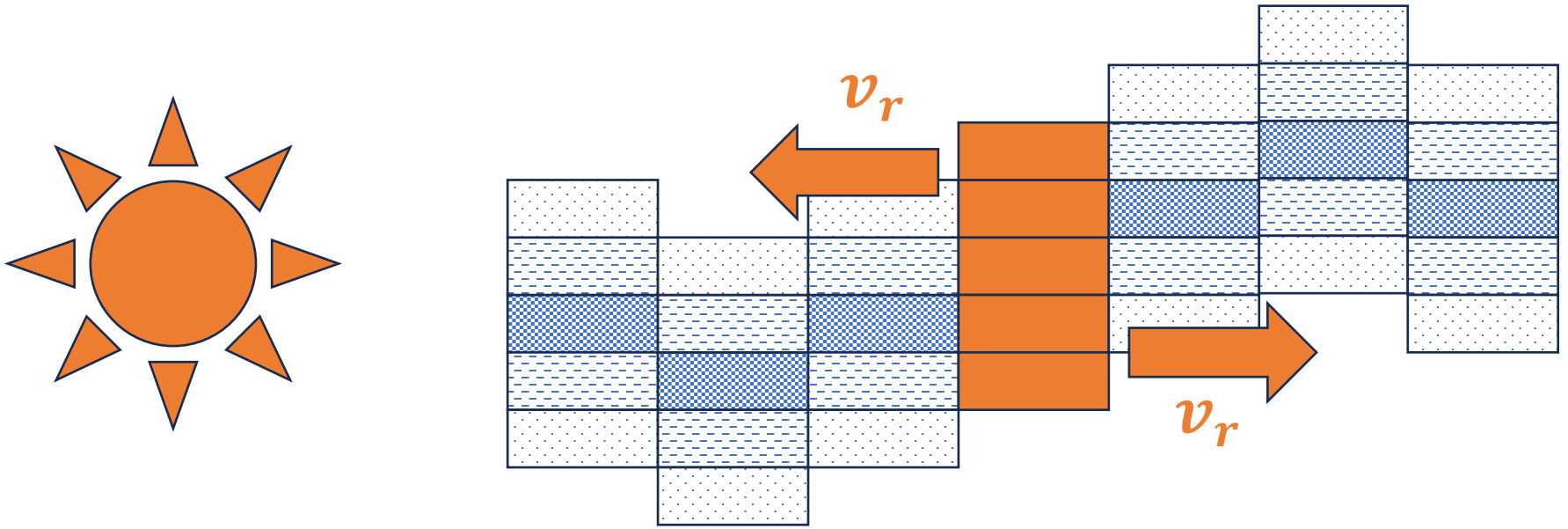


1. $\vec{G} = \int v_r \rho (\vec{r} \times \vec{v}) r^2 d\Omega$
   Flux / Internal torque

2. $\partial_t \hat{l} = \partial_r \vec{G}$
   analogous to $\partial_t \Sigma = \partial_r \dot{M}$

See also Papaloizou & Lin (1995), Ogilvie (1999), Lubow & Ogilvie (2000), Ogilvie & Latter (2013), Foucart & Lai (2014), Dullemond+ (2022), Deng & Ogilvie (2022), etc., for more about theories

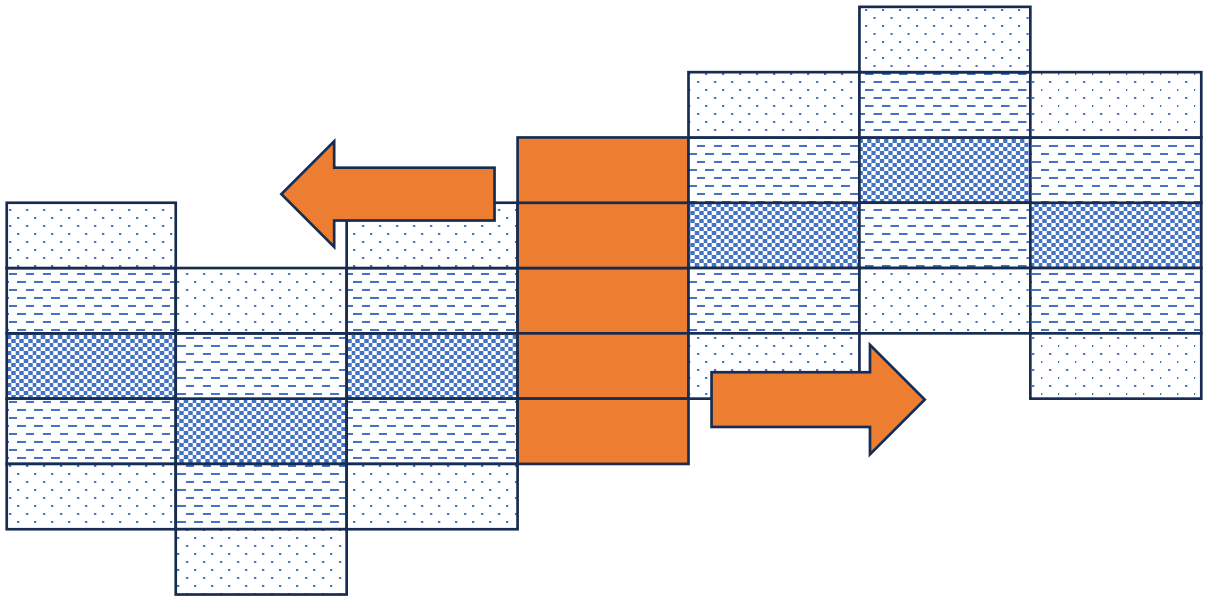
Warp Dynamics: Two Coupled Pieces of Physics



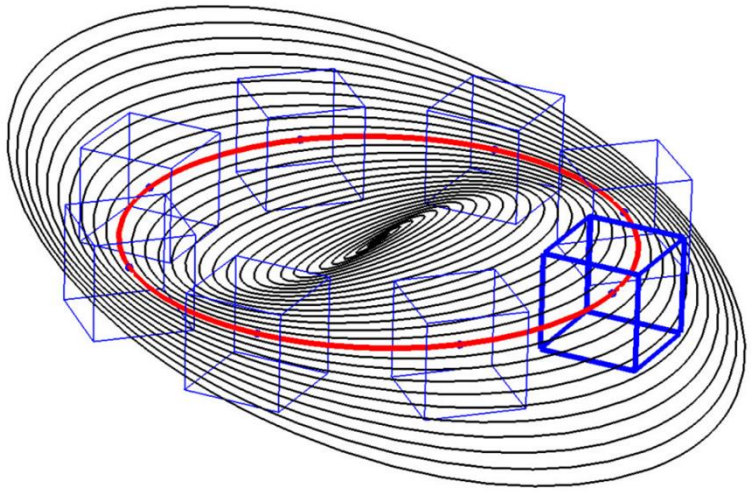
1. Warp (ψ) produces radial sloshing motion (v_r).
2. Sloshing (v_r) transports angular momentum and shapes the warp.

See also Papaloizou & Lin (1995), Ogilvie (1999), Lubow & Ogilvie (2000), Ogilvie & Latter (2013), Foucart & Lai (2014), Dullemond+ (2022), Deng & Ogilvie (2022), etc., for more about theories

Warp Dynamics: Two Coupled Pieces of Physics



Adapted from Ogilvie (2022)



Resonance drives strong sloshing!

$$U + 2iV = \frac{\psi}{\alpha} e^{-i\phi}$$

1. Warp (ψ) produces radial sloshing motion (v_r).
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Questions and Our Setup

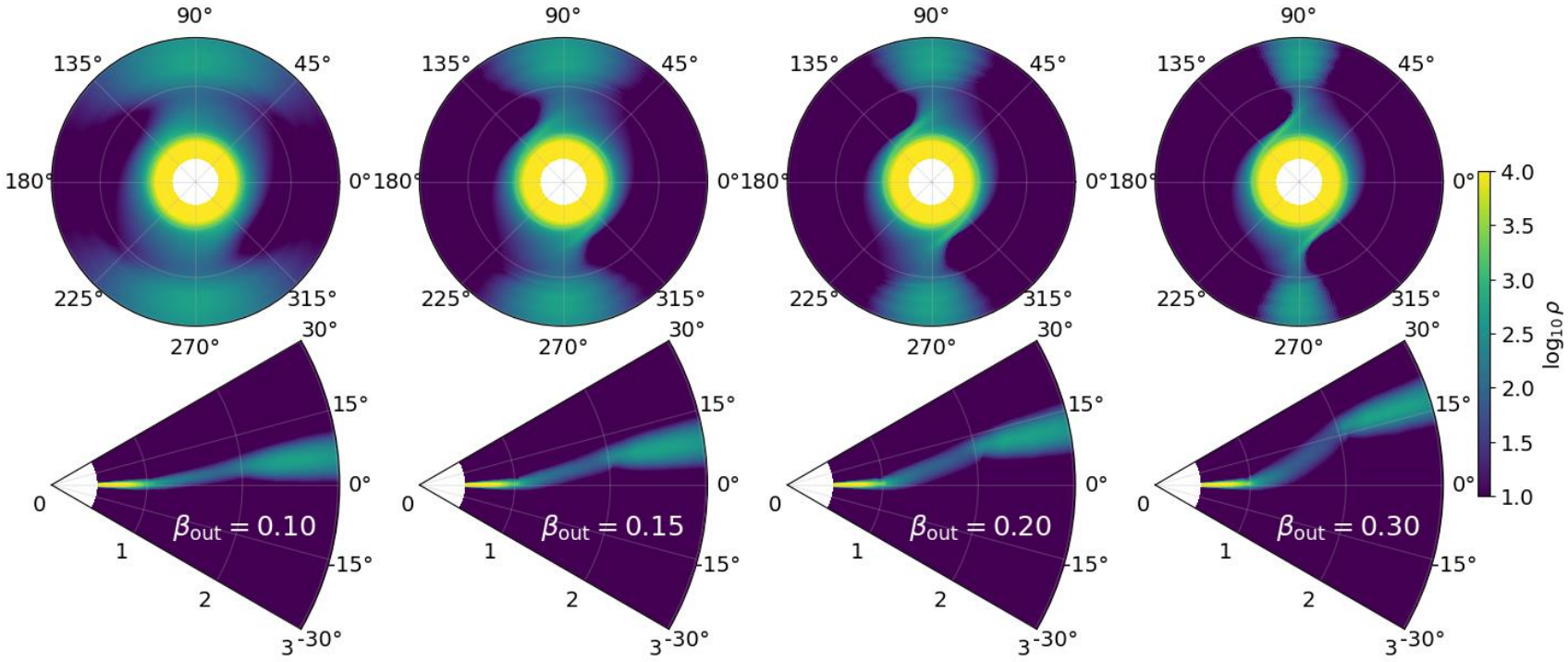
Q1: How do disks bend and sustain coherent warps?
Q2: Under what conditions do they break into misaligned components?

Our Approach:

- ***Focus on internal hydrodynamics***

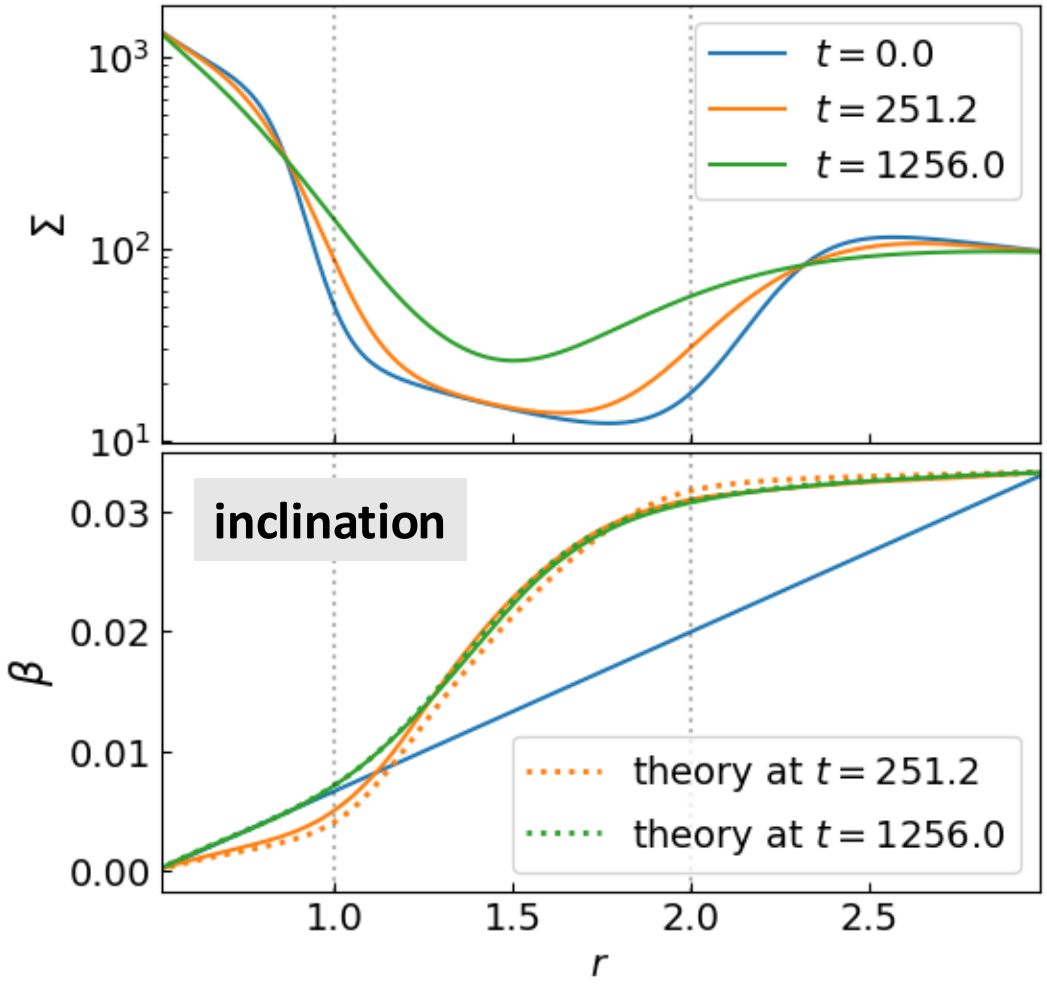
- **Hold** the edges of the disk at **fixed** inclinations

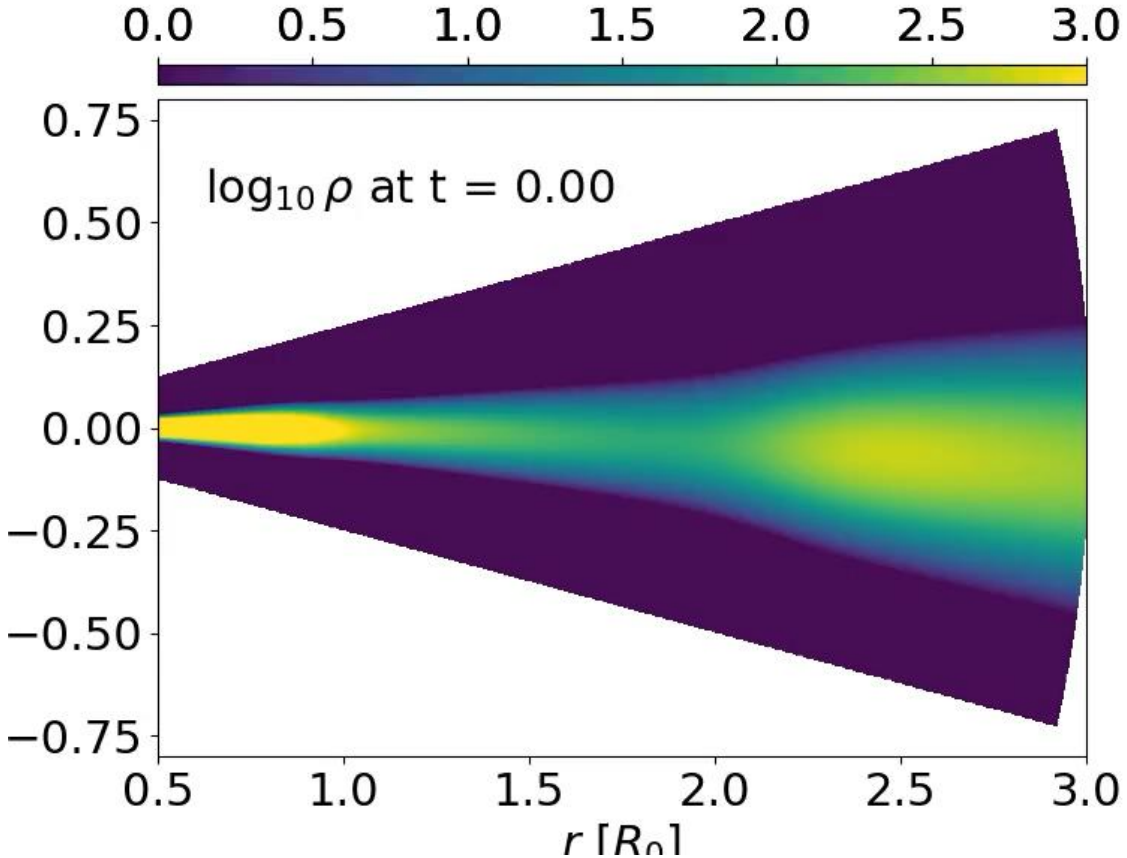
- **Theory** + **3D hydrodynamic simulations (Athena++)**



See also Papaloizou & Lin (1995), Ogilvie (1999), Lubow & Ogilvie (2000), Ogilvie & Latter (2013), Foucart & Lai (2014), Dullemond+ (2022), etc., for more about theories
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Small Warp – Linear Regime

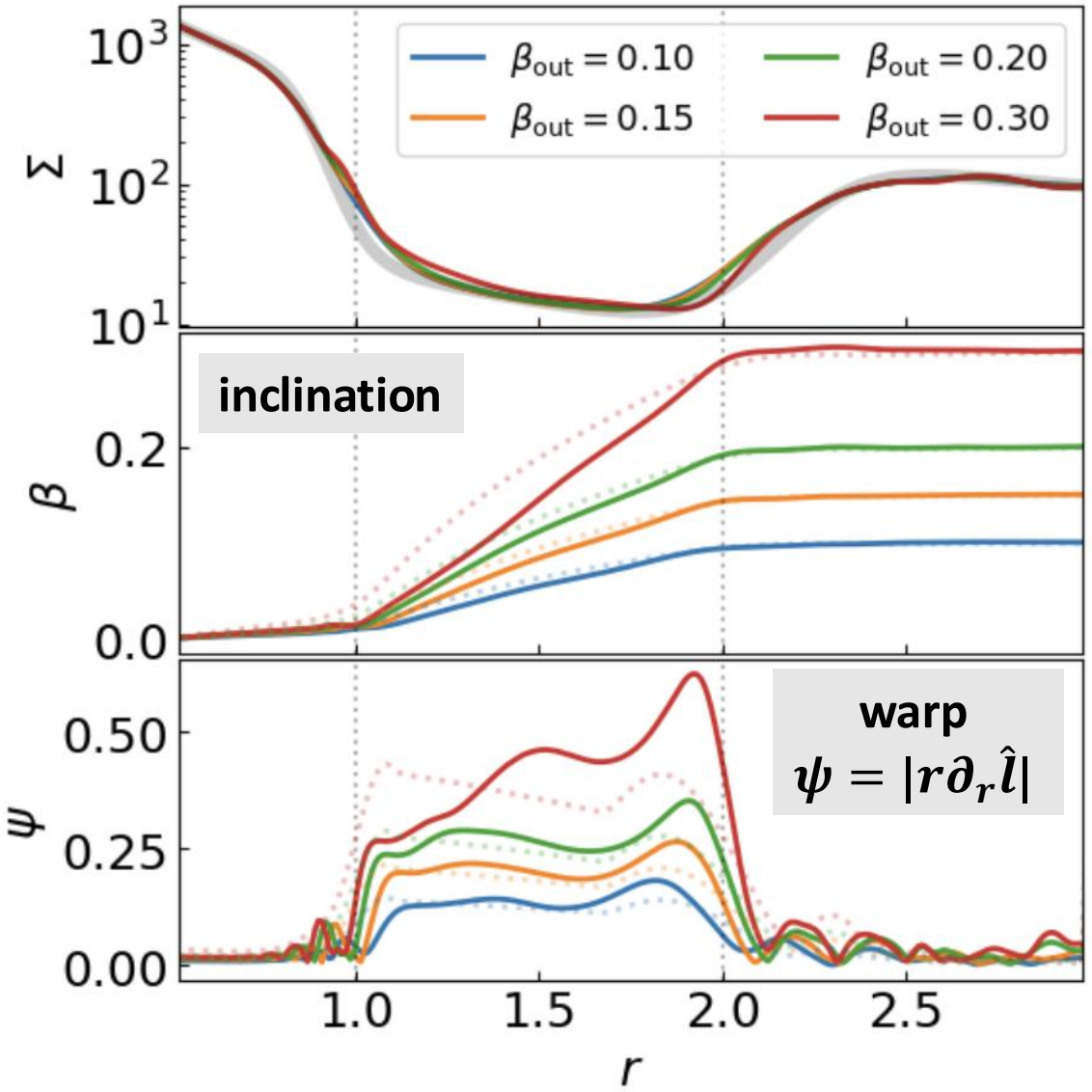


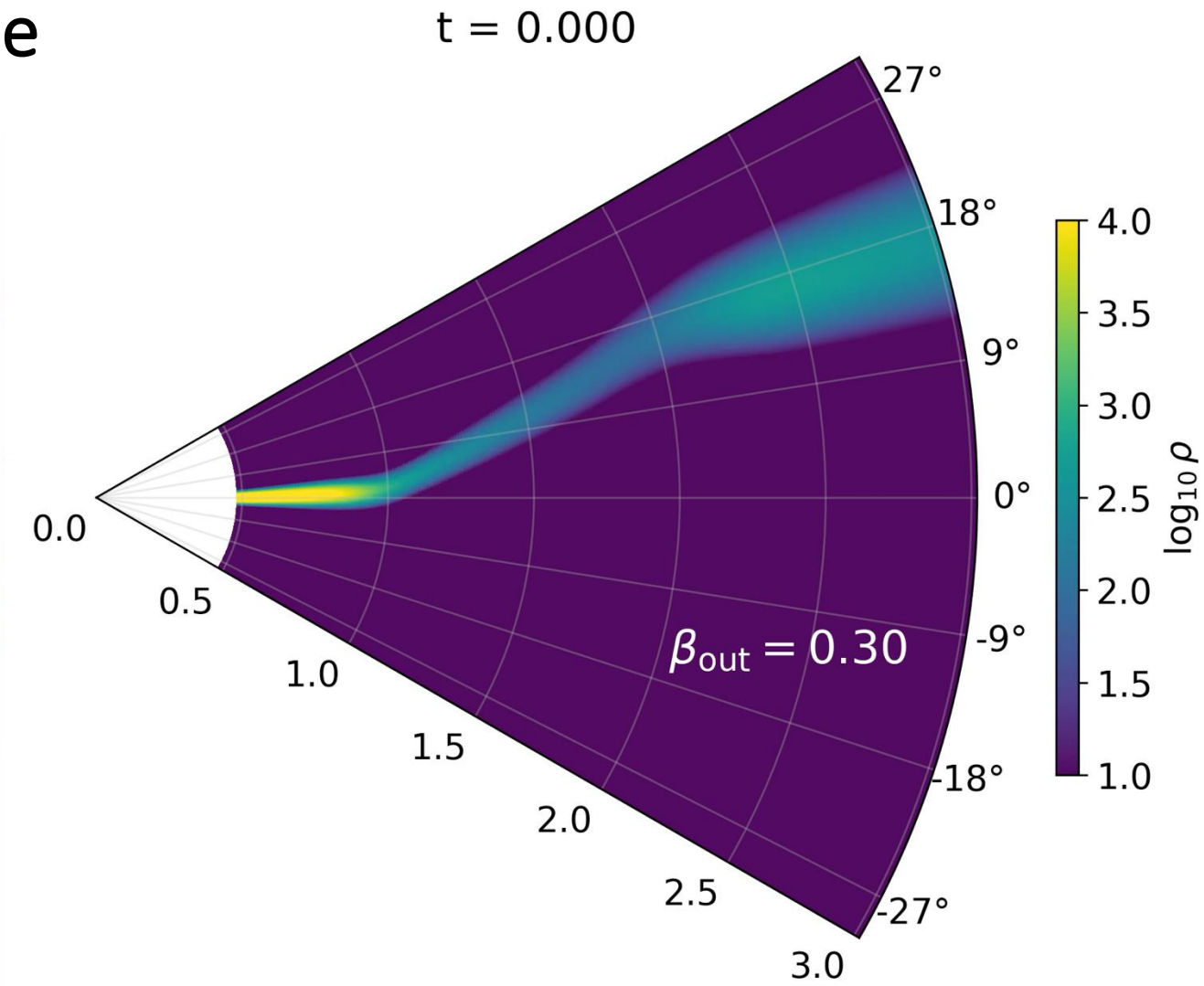


When the $\beta_{out} - \beta_{in}$ is **small**:

- Disk relaxes into a **steady state** on a short timescale.
- Simulation agrees with the linear theory (*Papaloizou & Lin, 1995; Lubow & Ogilvie, 2000*)

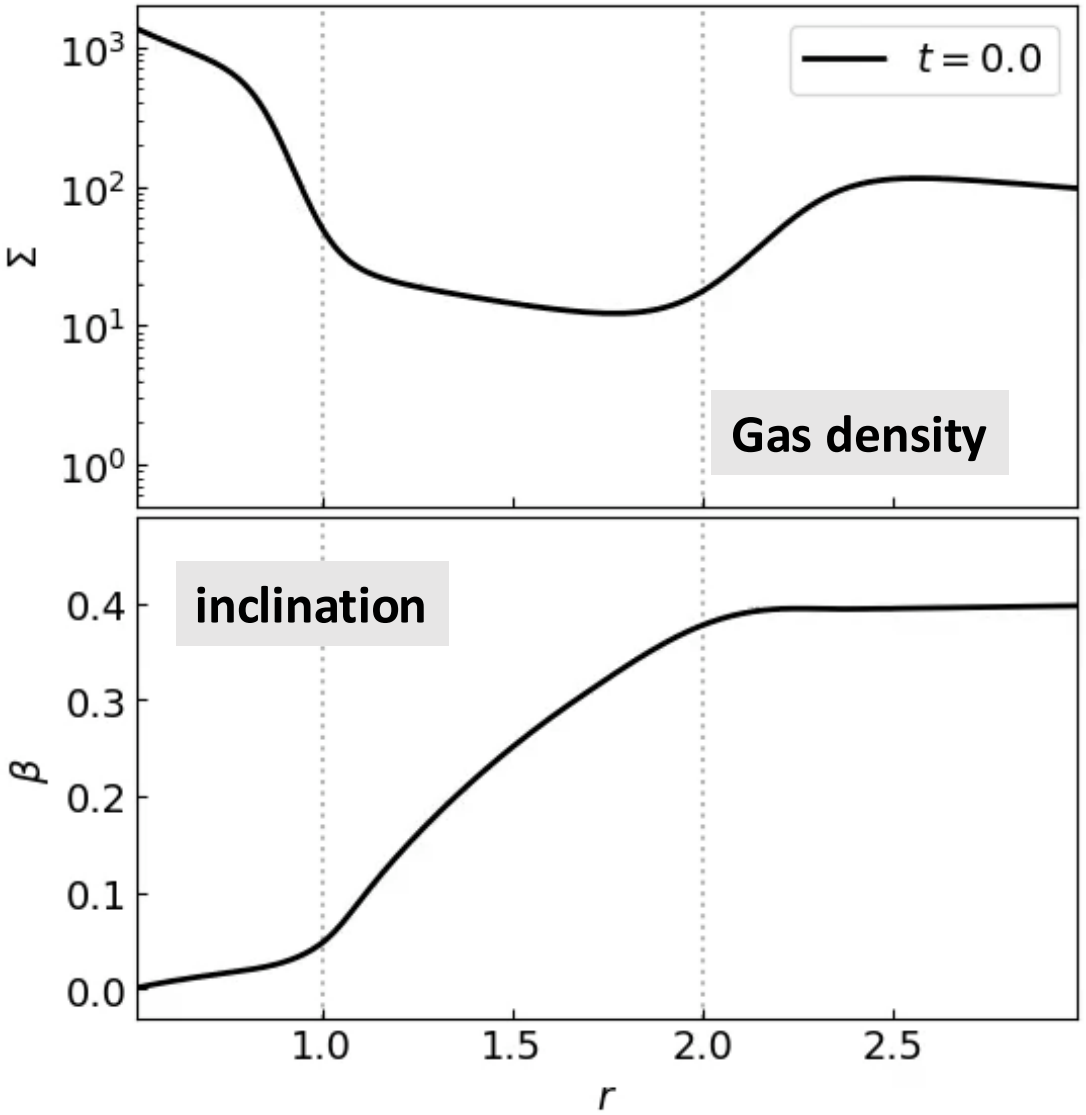
Strong Warp– Nonlinear Regime

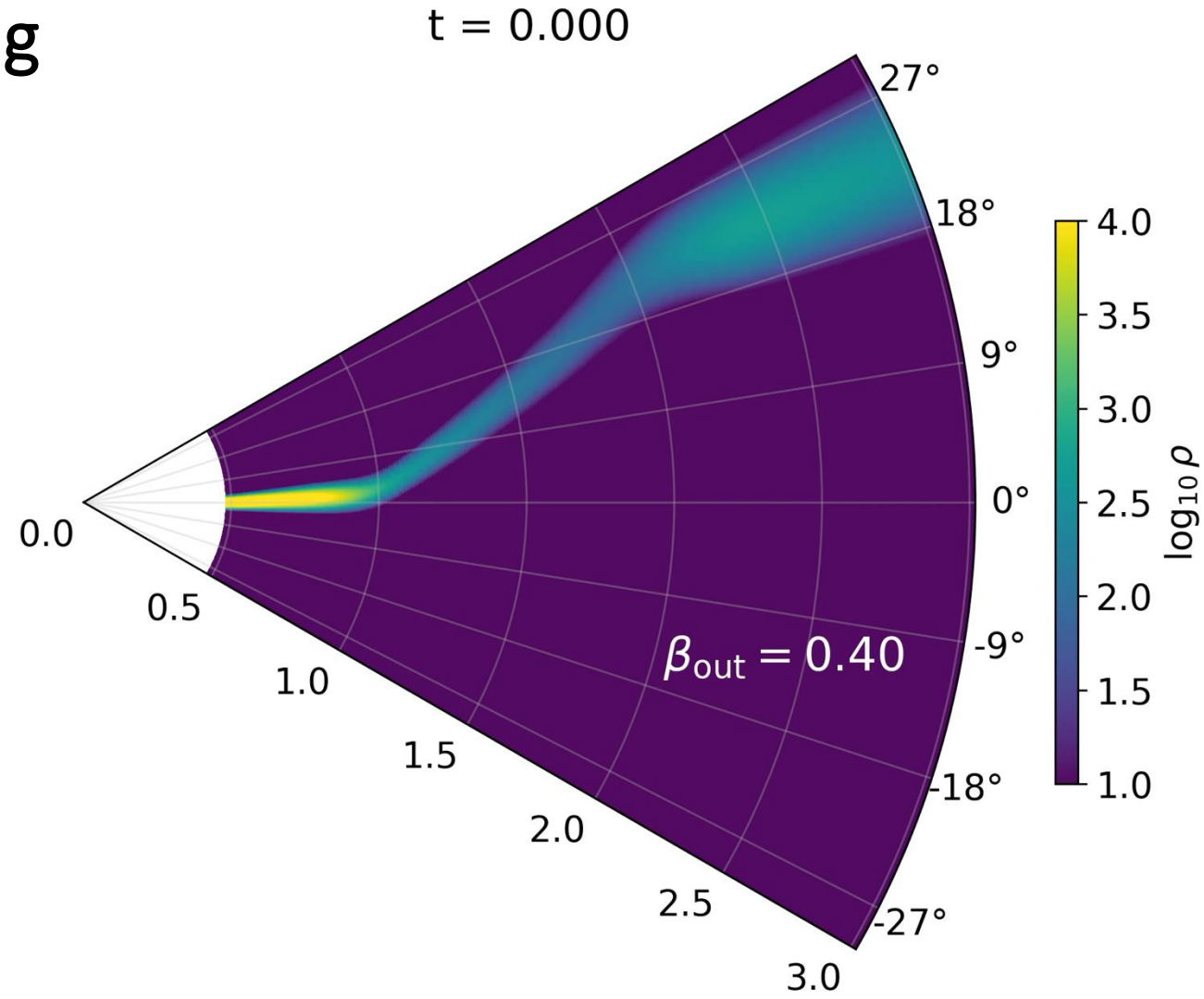




Simulation with $\boldsymbol{\beta_{out} = 0.3}$

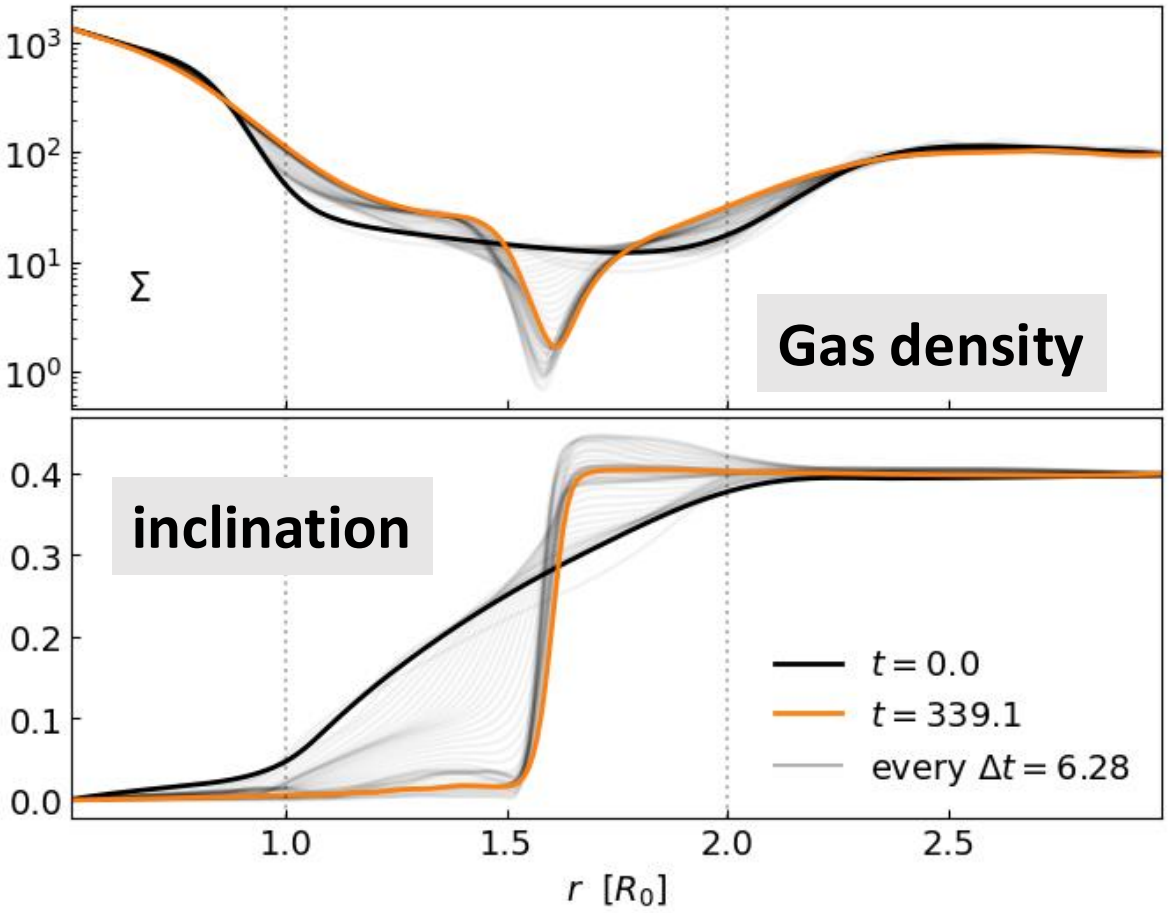
Even Stronger Warp: Disk Breaking

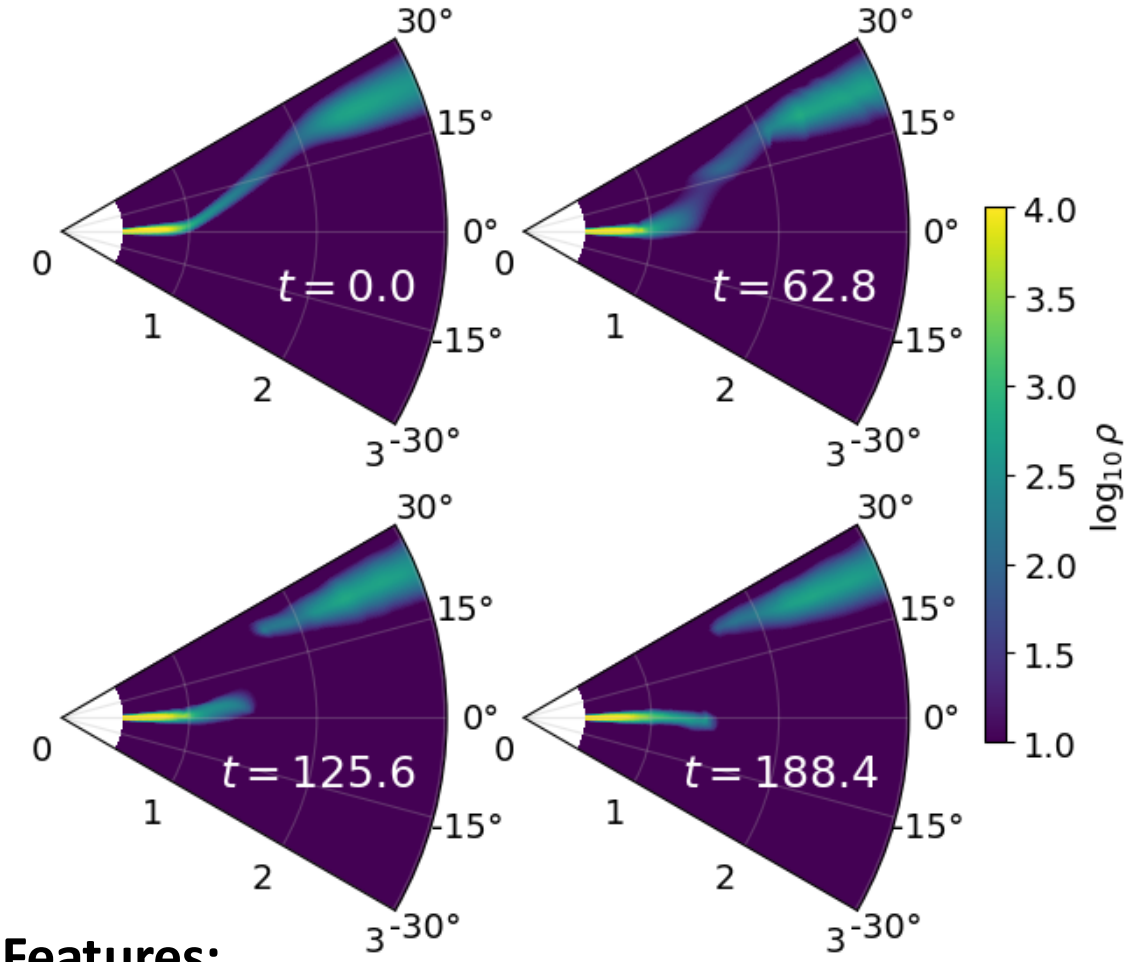




Simulation with $\beta_{out} = 0.4$

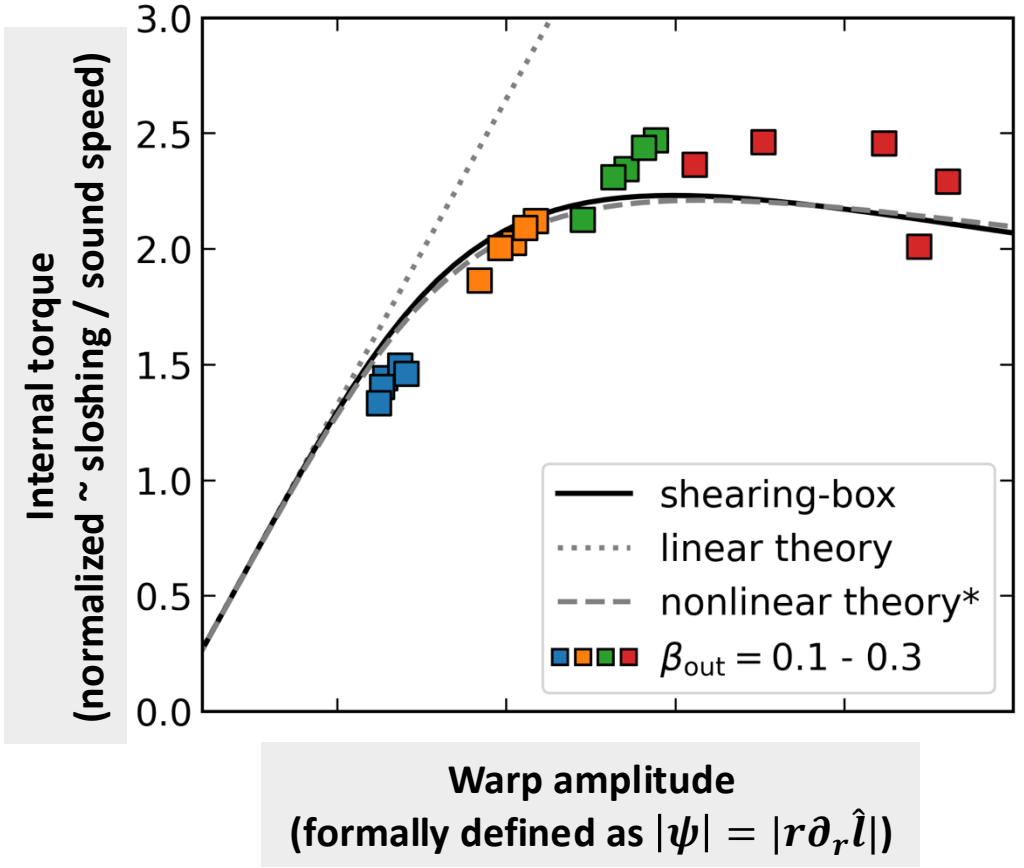
Even Stronger Warp: Disk Breaking





Features:

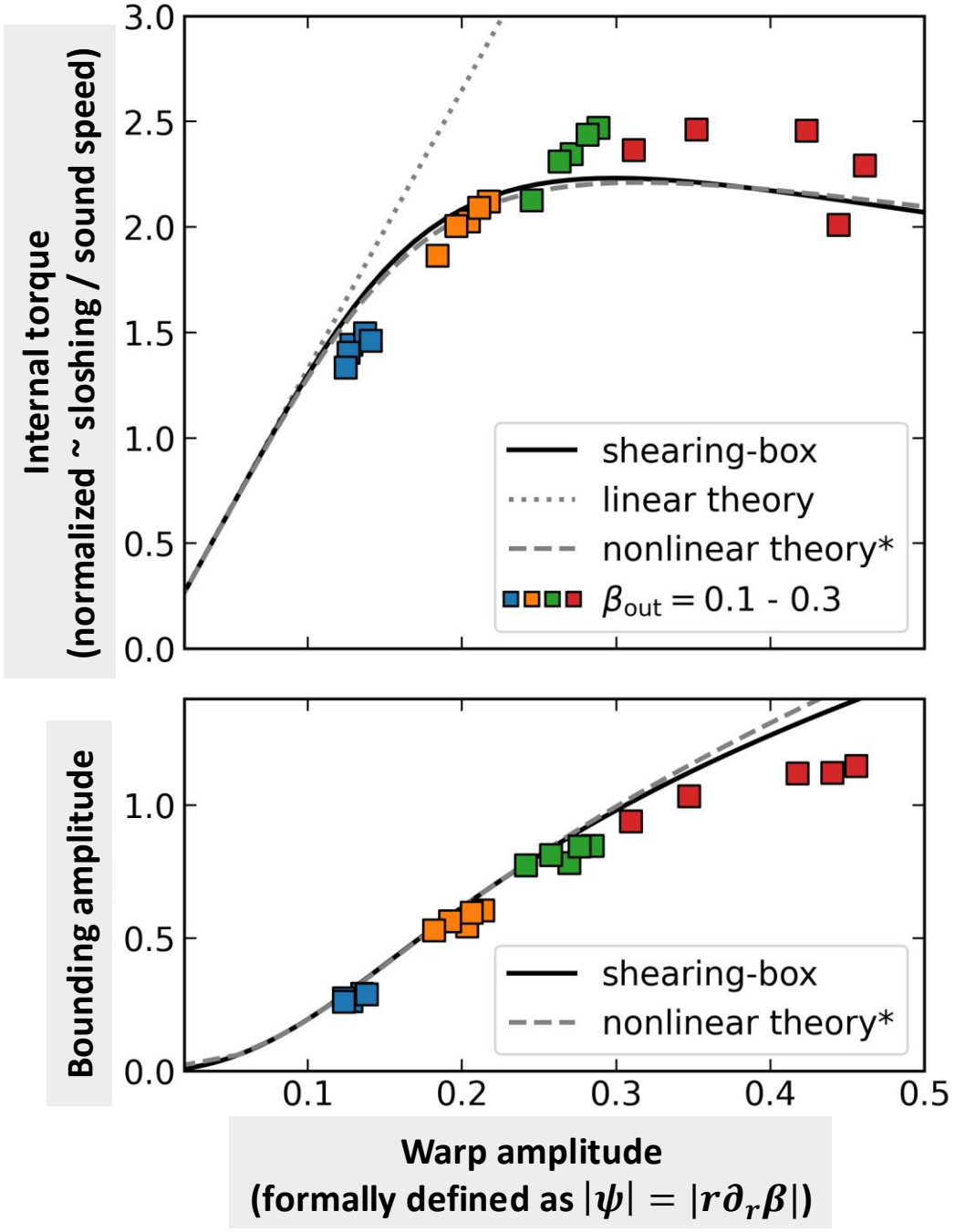
1. **Density drops** at the middle of the disk.
2. **The inclination profile** becomes a step function, representing a broken disk with two misaligned parts..

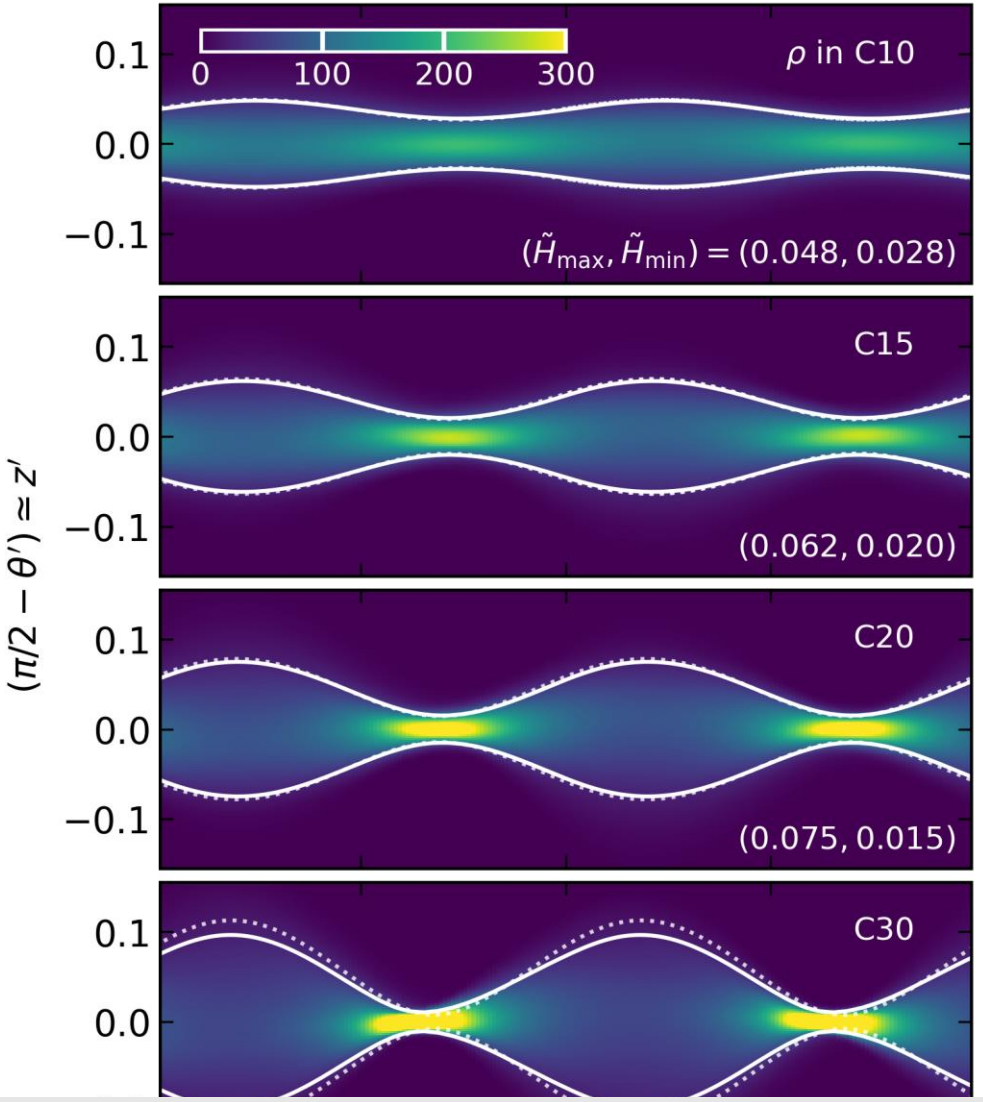


← internal torque saturates

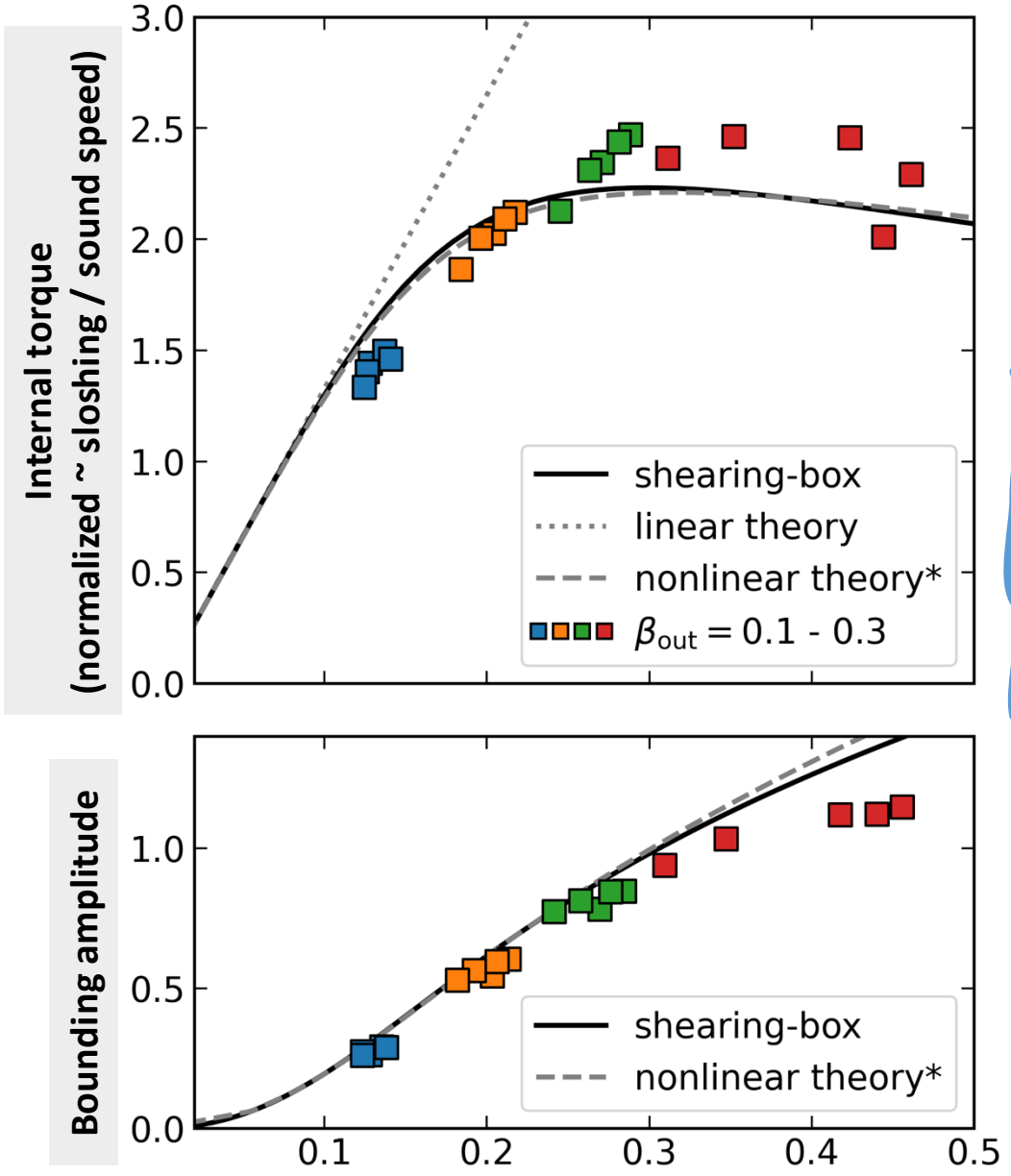
$$\vec{G} = \int v_r \rho (\vec{r} \times \vec{v}) r^2 d\Omega$$

measured at several r's,
then normalized by local $\{r, \Sigma, \Omega, H\}$





Bouncing: gas undergoes extreme vertical compression and expansion.



Bouncing causes the saturation.

$$\eta \equiv \frac{H(\phi)}{H_0}$$

$$\partial_\phi U - 2V = \frac{|\psi|\cos\phi}{\eta} - \frac{\alpha}{3}\frac{|\psi|\cos\phi}{\eta^2}(W + |\psi|\cos\phi U) - \alpha\left[\frac{|\psi|\sin\phi}{\eta} + \frac{\left(1+|\psi|^2\cos^2\phi\right)U}{\eta^2}\right],$$

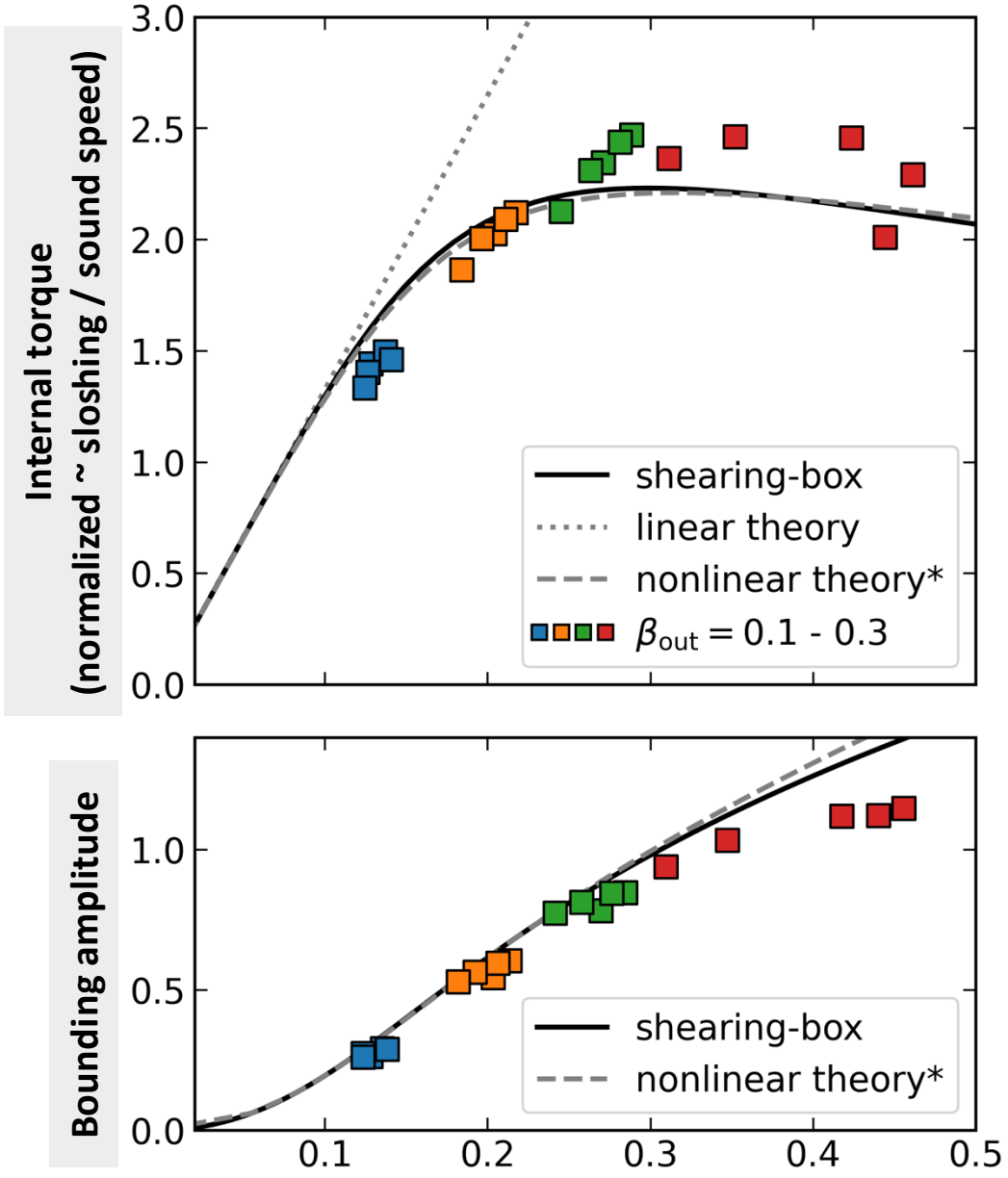
$$\partial_\phi V + \frac{1}{2}U = \frac{3}{2}\alpha\frac{|\psi|\cos\phi}{\eta} - \alpha\frac{\left(1+|\psi|^2\cos^2\phi\right)V}{\eta^2},$$

$$\partial_\phi W + |\psi|\sin\phi U = \frac{1}{\eta} - \eta - \frac{\alpha}{3}\frac{W + |\psi|\cos\phi U}{\eta^2} - \alpha\left[\frac{|\psi|^2\sin\phi\cos\phi}{\eta} + \frac{\left(1+|\psi|^2\cos^2\phi\right)W}{\eta^2}\right],$$

$$\partial_\phi \eta = W + |\psi|\cos\phi U,$$

Nonlinear shearing-box theory (Ogilvie & Latter 2013)

\+ accurate
\- no analytical solution
\- hard to interpret



Bouncing causes the saturation.

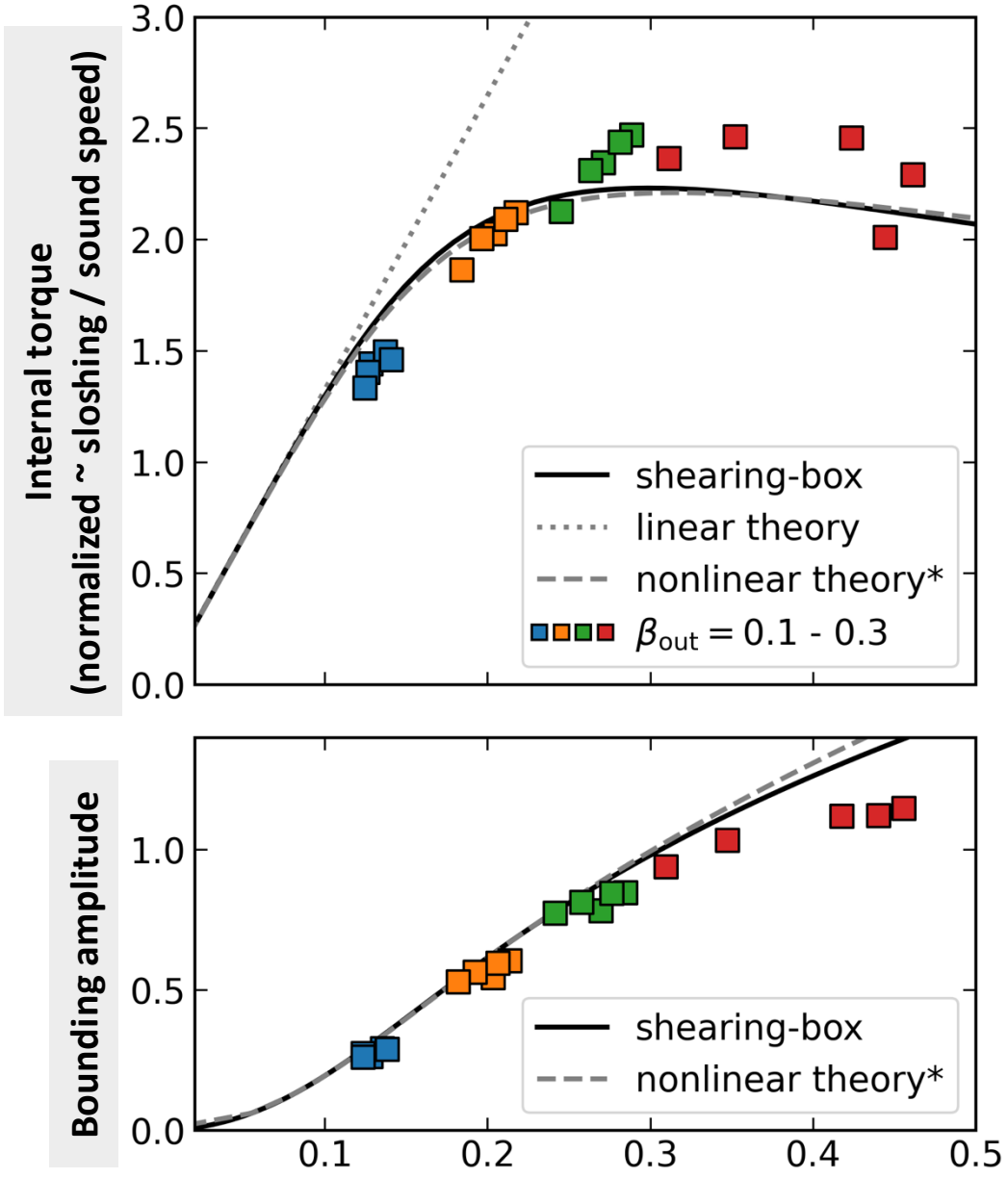
$$\eta \equiv \frac{H(\phi)}{H_0}$$

neglect bouncing $\eta = 1$

$$\partial_\phi U - 2V = \psi cos\phi - \alpha U$$
$$\partial_\phi V + \frac{1}{2}U = -\alpha V$$

include bouncing $\eta \neq 1$

$$\partial_\phi U - 2V = \psi cos\phi \eta^{-1} - \alpha U \eta^{-2}$$
$$\partial_\phi V + \frac{1}{2}U = -\alpha V \eta^{-2}$$



Bouncing causes the saturation.

Simplified nonlinear theory (ours)

$$\partial_\phi \mathcal{E} + \left(i + \alpha\eta^{-2}\right)\mathcal{E} = \alpha\cos\phi\,\eta^{-2}$$

$$\partial_\phi^2\eta - \eta^{-1} + \eta = \frac{\psi^2}{\alpha}\left[-2\sin\phi\,\Re(\mathcal{E}) + \cos\phi\,\Re\left(\partial_\phi\mathcal{E}\right)\right]$$

sloshing $(m=-1)$: $\mathcal{E} \equiv U + 2iV = A\,e^{-i(\phi+\phi_0)}$

bouncing $(m=\pm1)$: $\eta = B\,[1 + \Delta\sin(2\phi + \phi_0)]$

$$A = \sqrt{\frac{3-2p^2}{3}}\frac{\left(1-p^2\right)^{3/2}}{2(1+p^2)},$$

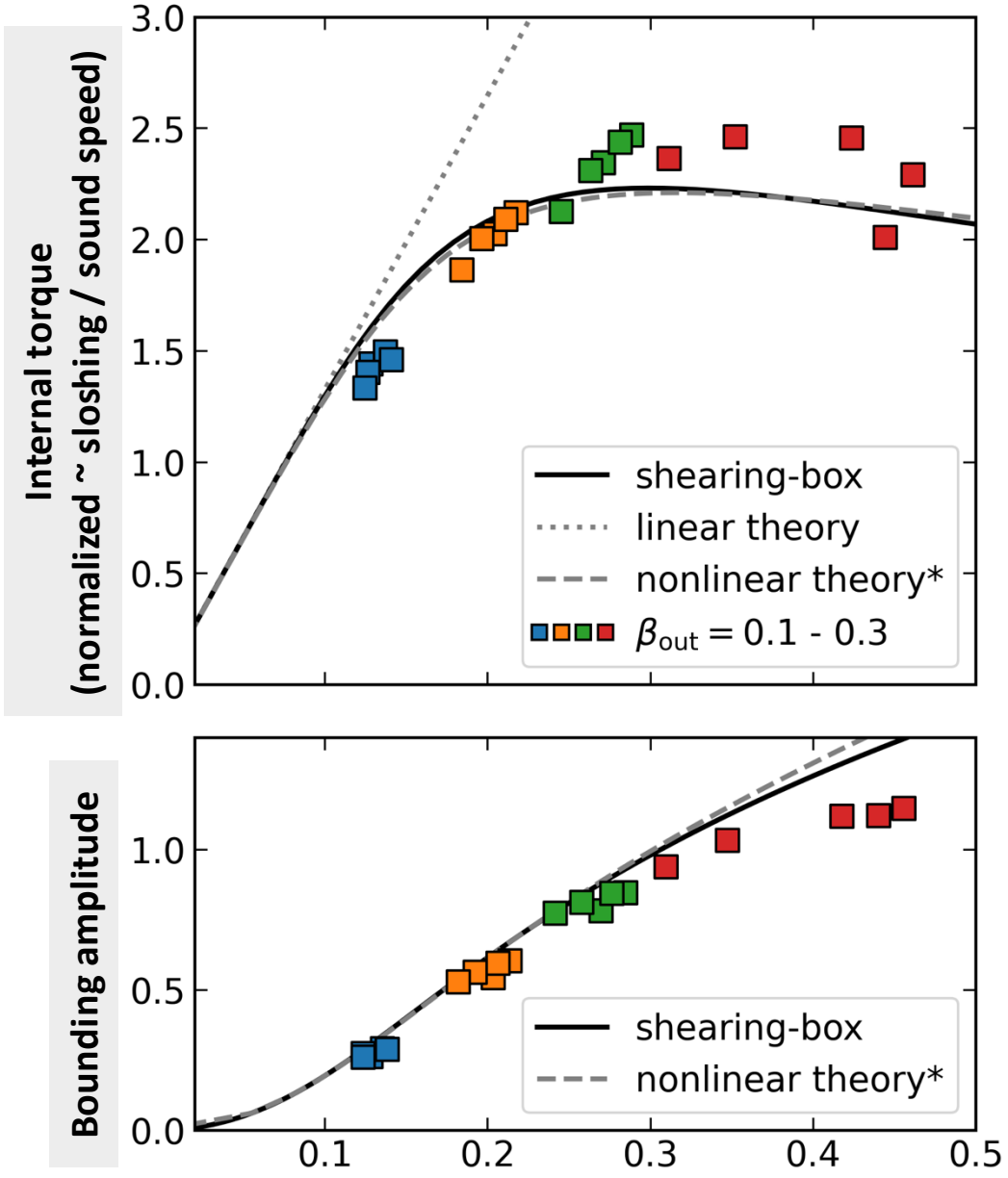
$$B = \sqrt{\frac{3-2p^2}{3}}\frac{1+p^2}{1-p^2}$$

$$\phi_0 = \sin^{-1}p, \qquad \Delta = \frac{2p}{1+p^2}$$

+ accurate
+ has analytical solution

$$p \simeq \frac{\psi^2/\alpha}{4.6 + 1.35(\psi^2/\alpha)}$$

Dashed curve ----



Bouncing causes the saturation.

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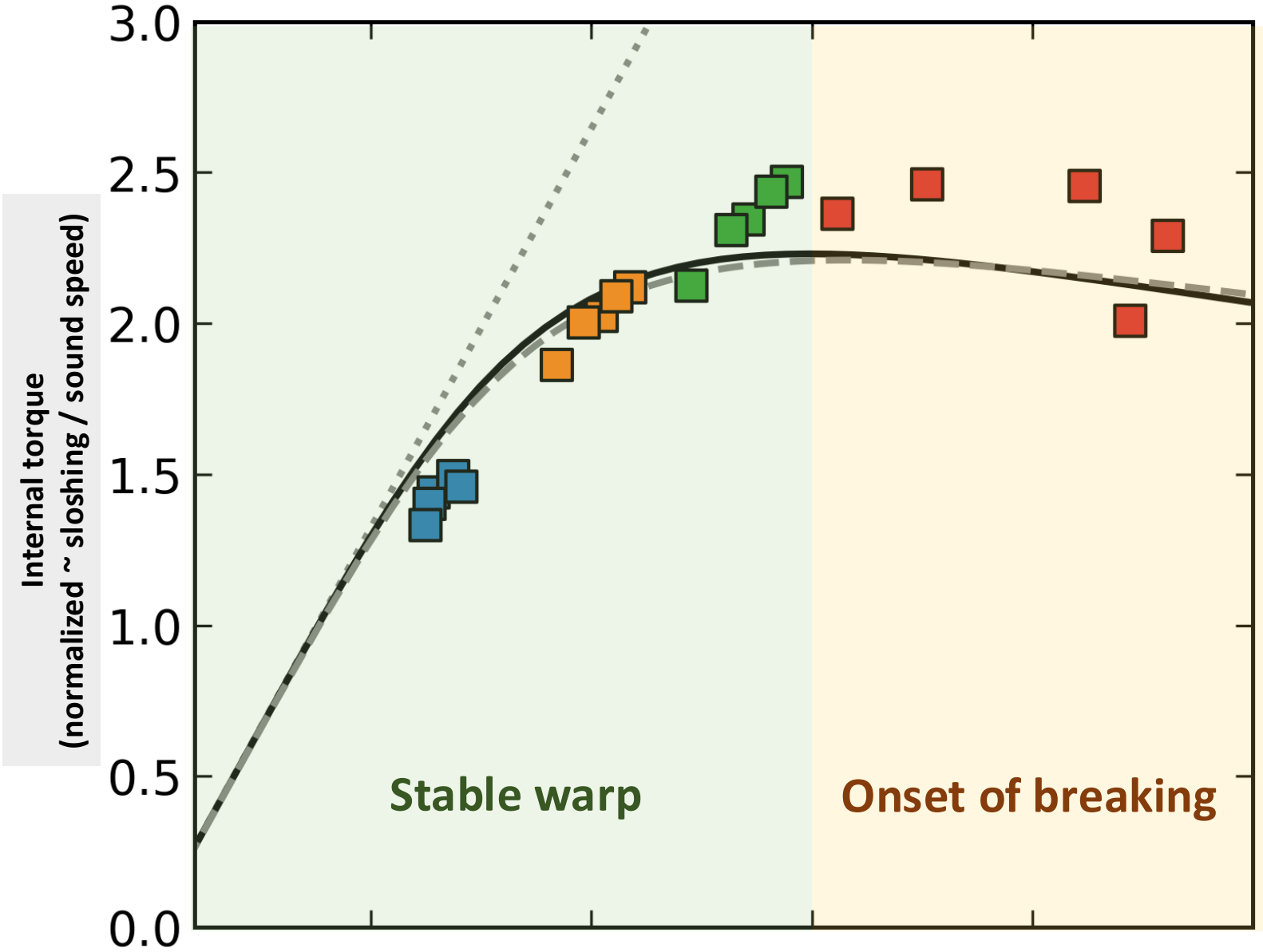
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less m=1 forcing: modulation (↘)

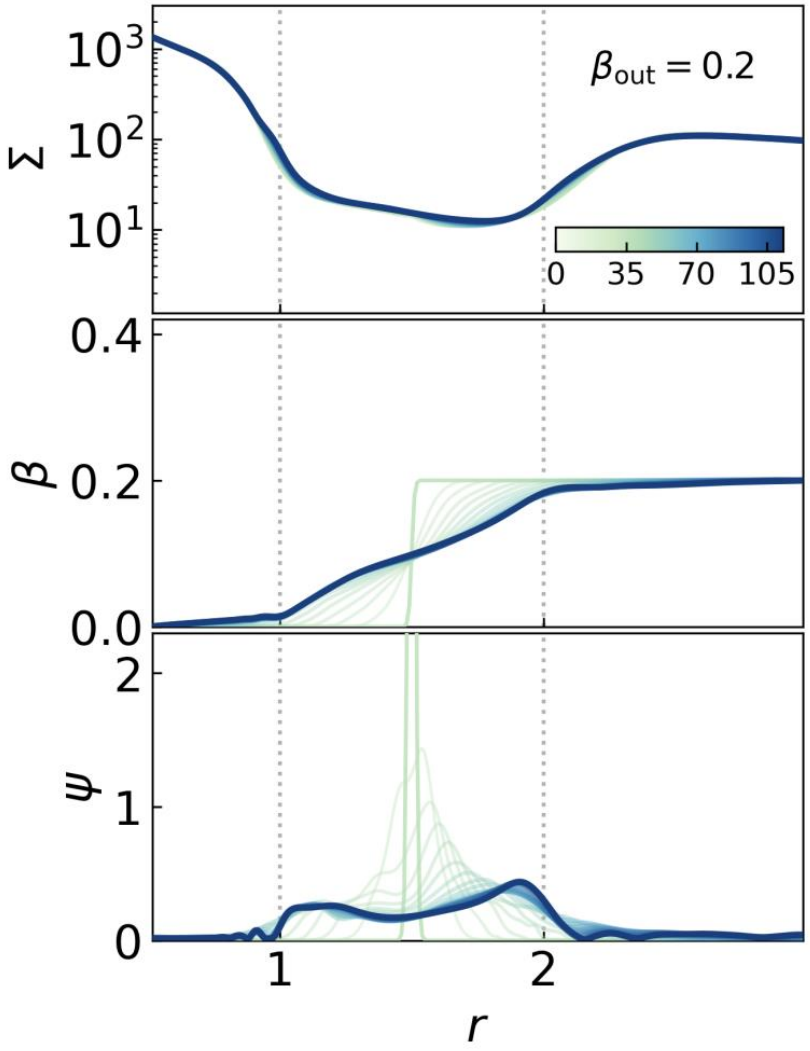
$$U + 2iV = \frac{\psi}{\alpha}e^{-i\phi}\frac{\left\langle e^{i\phi}\cos\phi\,\eta^{-1}\right\rangle}{\left\langle\eta^{-2}\right\rangle}$$

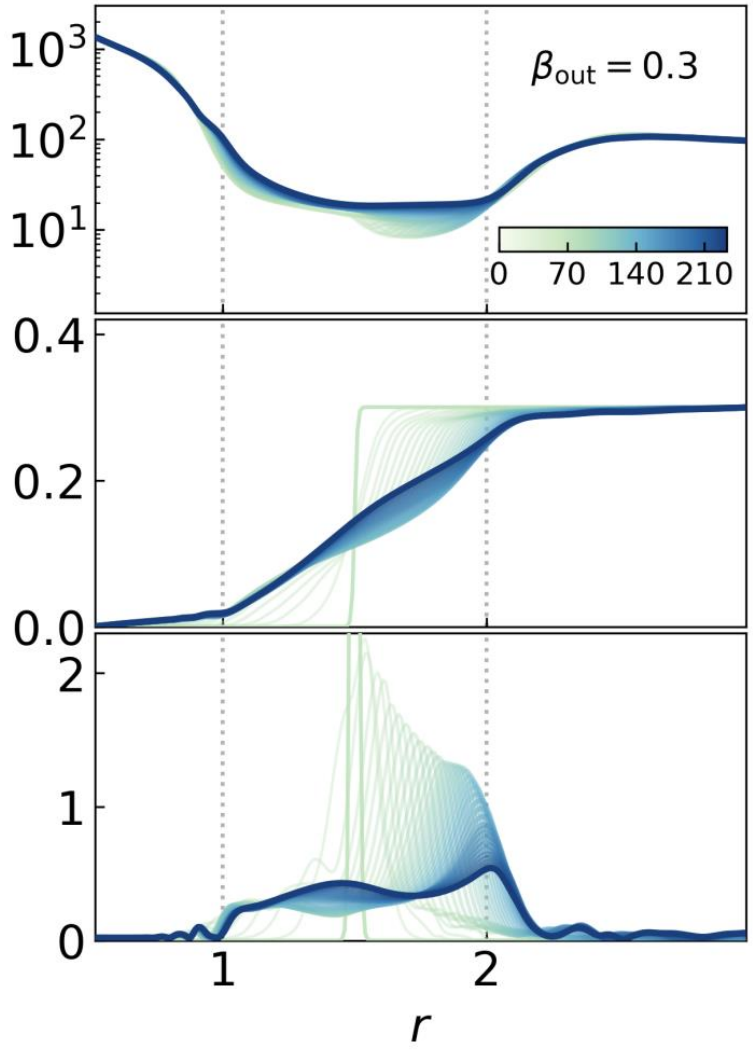
more viscosity effect: detuning (↗)

Thus, sloshing and the internal torque are weakened.

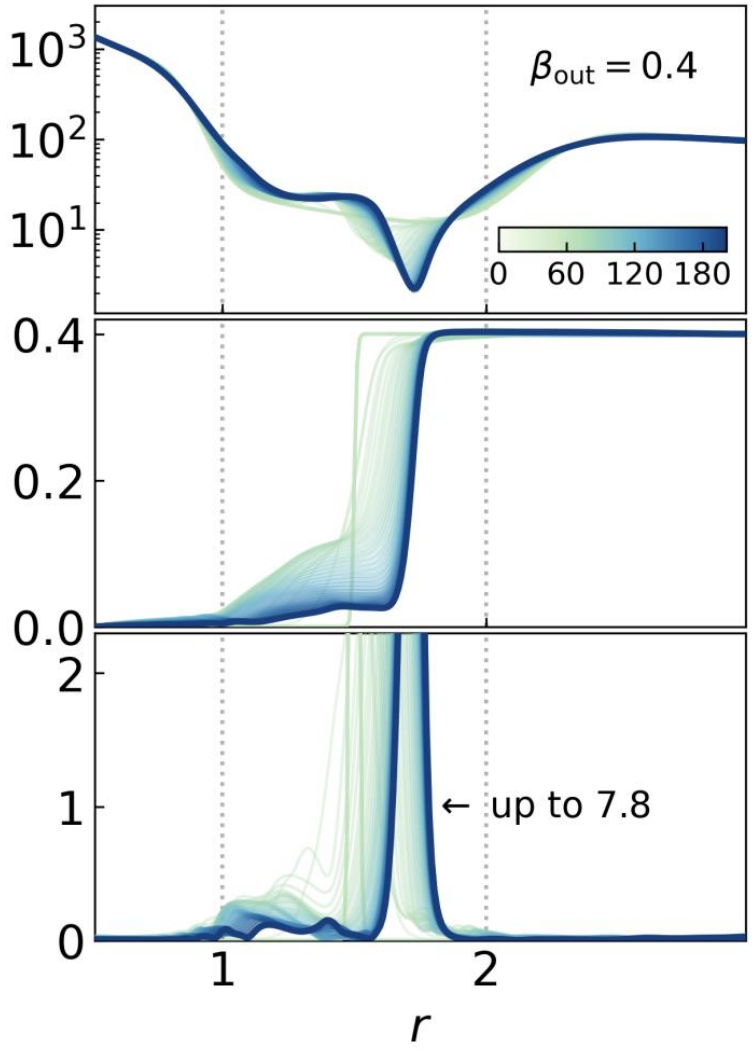

3.0
2.5
2.0
1.5
1.0
0.5
0.0
Internal torque
(normalized ~ sloshing / sound speed)
Stable warp
Onset of breaking
Warp amplitude
(formally defined as $|\psi| = |r\partial_r\beta|$)

Broken disks can heal if β_{out} is small.





But not when β_{out} is large.



Summary

- Warped disks are dynamical systems. **Internal torque** plays an important role.
  - **Q1:** Disks sustain coherent warps by generating and balancing their internal torque.
  - **Q2:** Breaking triggers when internal torque over saturates.

- **Bouncing**, i.e. the azimuthal variation of the disk density scale height, causes the **saturation of the internal torque** in strongly warped disks.

- Our simulations show **good agreements with theories** (e.g., *Papaloizou & Lin 1995, Lubow & Ogilvie 2000, Ogilvie & Latter 2013*).

Summary

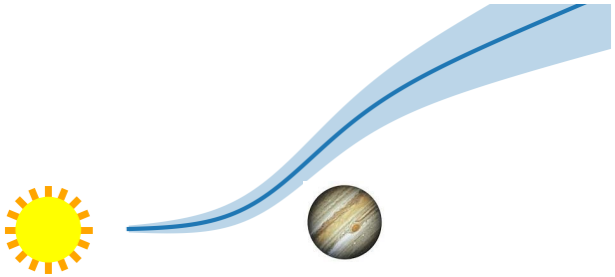
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Ph.D. student at CIERA



Check out ***Jupiter***'s poster and talk with him for planet-induced warps in protoplanetary disks.