Resonant Excitation of Planetary Eccentricity due to a Dispersing Eccentric Protoplanetary Disk

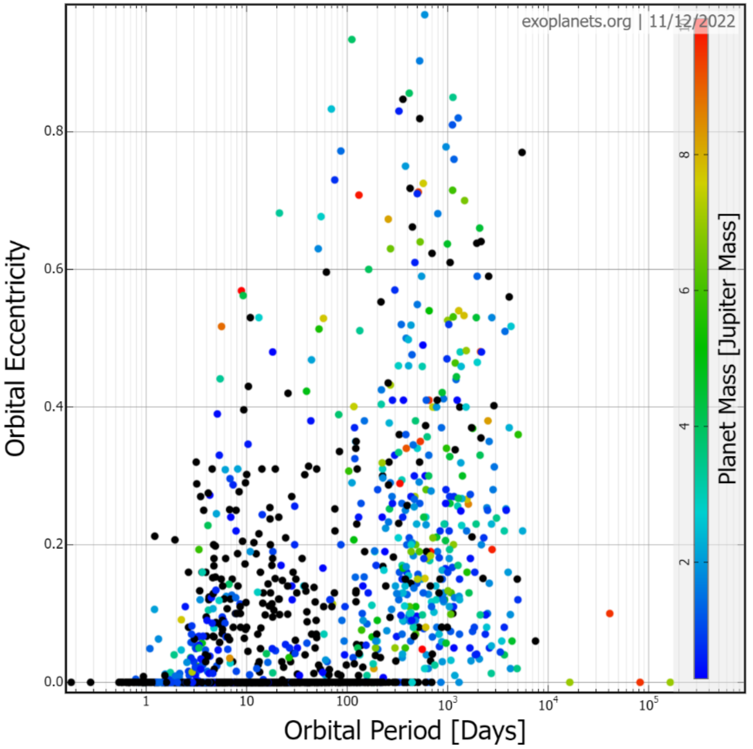
Jiaru Li (Cornell, advised by Prof. Dong Lai)

Nov. 28, 2022 @ Princeton

Orbital Eccentricity of Exoplanet

Possible origins:

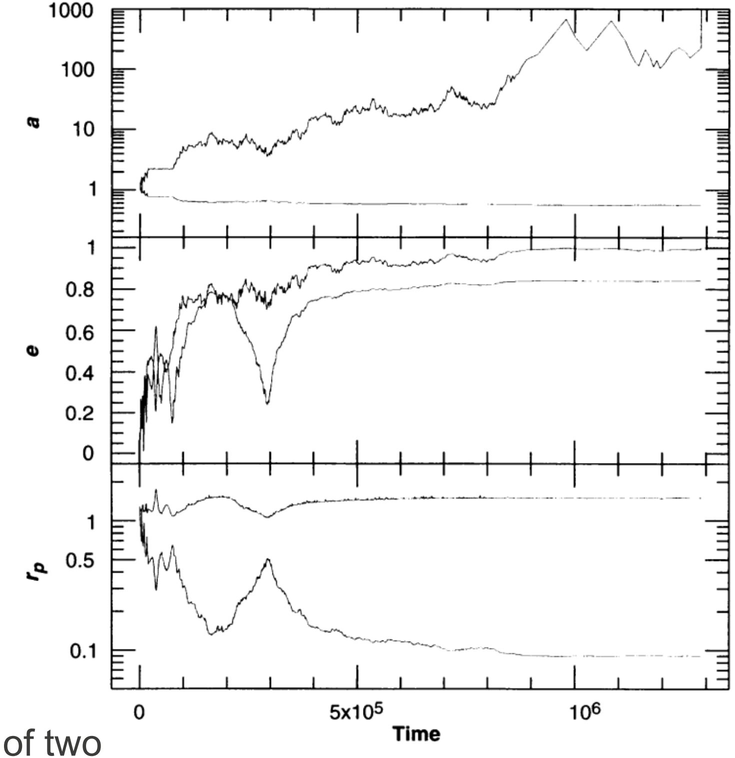
- Planet-planet scatterings
- Secular interactions with exterior companions
- Planet-disk interactions



Orbital Eccentricity of Exoplanet

Possible origins:

- **Planet-planet scatterings**
- Secular interactions with exterior companions
- Planet-disk interactions

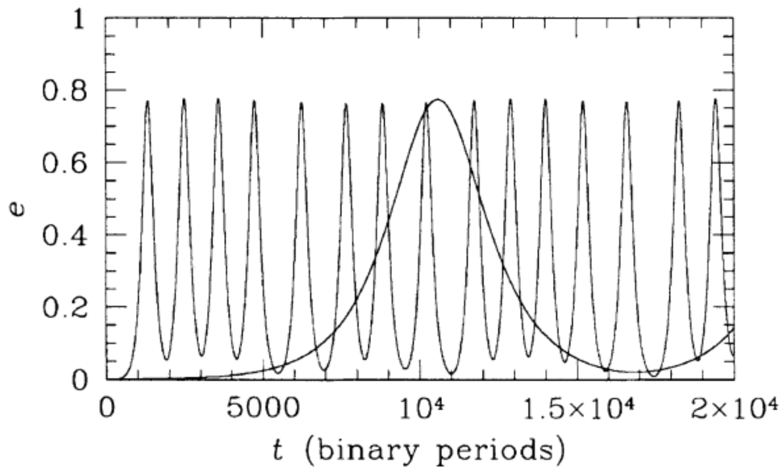


Time evolution of two planets undergoing scatterings (*Rasio & Ford 1996*)

Orbital Eccentricity of Exoplanet

Possible origins:

- Planet-planet scatterings
- **Secular interactions with exterior companions**
- Planet-disk interactions

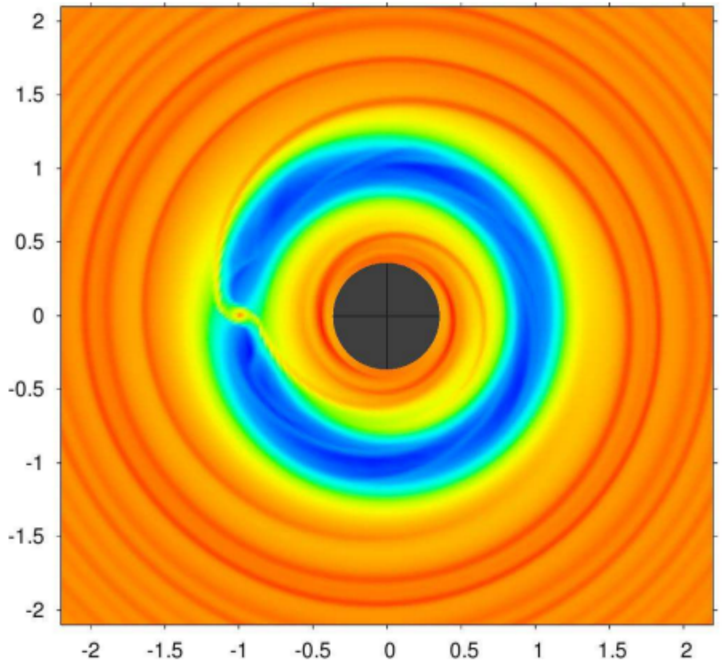


Eccentricity evolution of a planet undergoing Kozai oscillation (*Holmam+ 1996*)

Orbital Eccentricity of Exoplanet

Possible origins:

- Planet-planet scatterings
- Secular interactions with exterior companions
- **Planet-disk interactions**

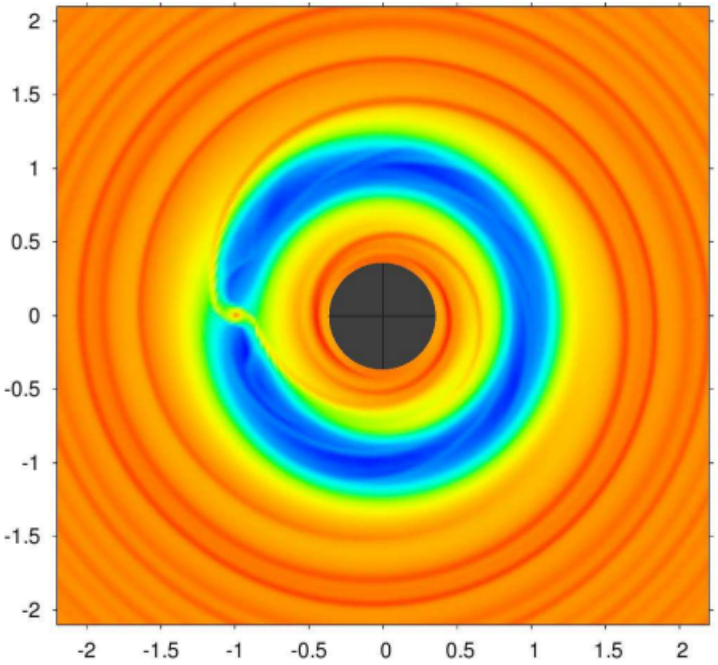


a disk and an embedd planet
(*Kley & Dirksen 2006*)

Orbital Eccentricity of Exoplanet

Possible origin:

- Planet-disk interactions
  - High mass planets: Lindblad torques (e.g., *Teyssandier & Ogilvie 2017*, *Ragusa et al. 2018*)
  - Low mass planets: thermal back-reactions (e.g., *Eklund & Masset 2017*, *Velasco Romero et al. 2022*)
  - However, in both cases, $e_p < 0.1$



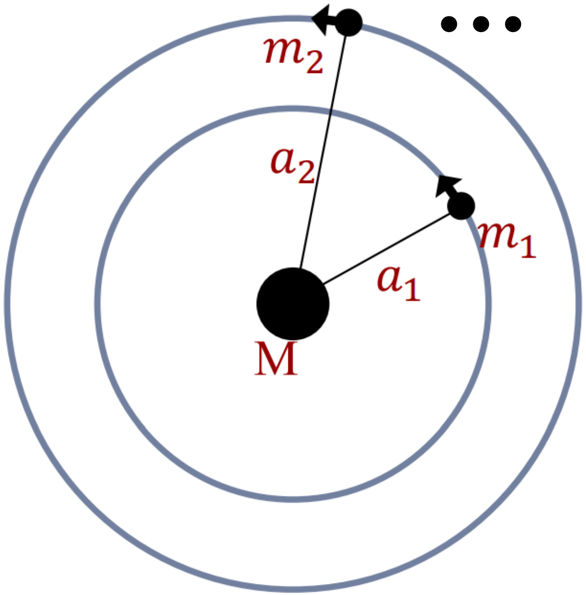
a disk and an embedd planet
(*Kley & Dirksen 2006*)

Agenda

- Planet-planet scatterings and tidal effects
- Theories of eccentric protoplanetary disks
- Resonant excitation of planetary eccentricity due to a dispersing eccentric disk

Part I: Planet-planet scatterings and tidal effects

Planet-planet scatterings



- Initial condition:

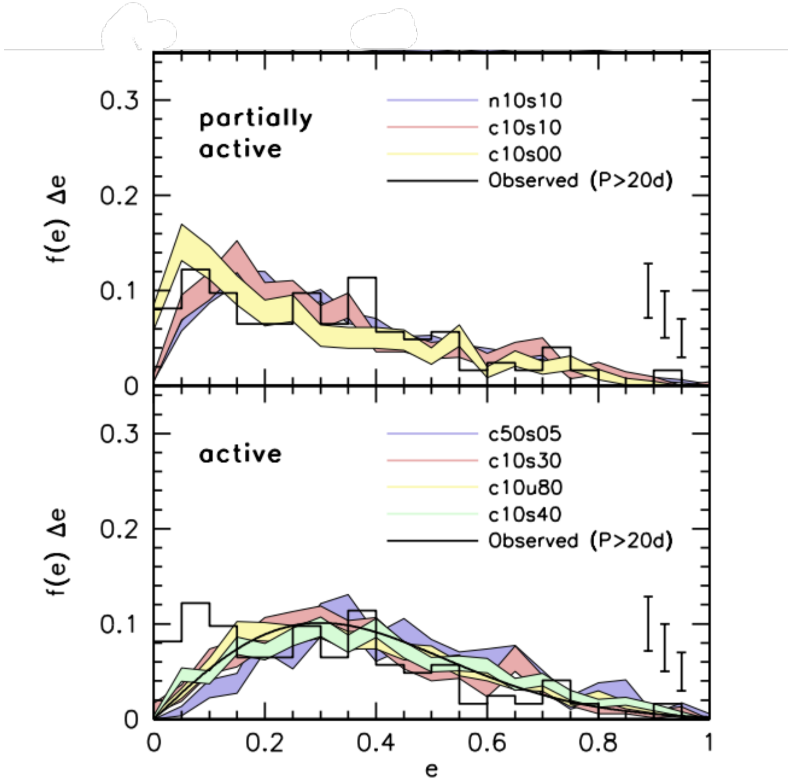
$$\boldsymbol{a_{i+1} - a_i \sim R_{\mathrm{H}}} \quad \text{where} \quad R_{\mathrm{H}} = \frac{a_i + a_{i+1}}{2}\left(\frac{m_i + m_{i+1}}{3M}\right)^{1/3}$$

(Dynamical instability will occur!)

- Reasons for such closely-packed orbits:
  - Planet formations in protoplanetary disks
  - Migration of planets

- One can N-body simulations to study the long-term outcomes.

Planetary eccentricity due to scatterings

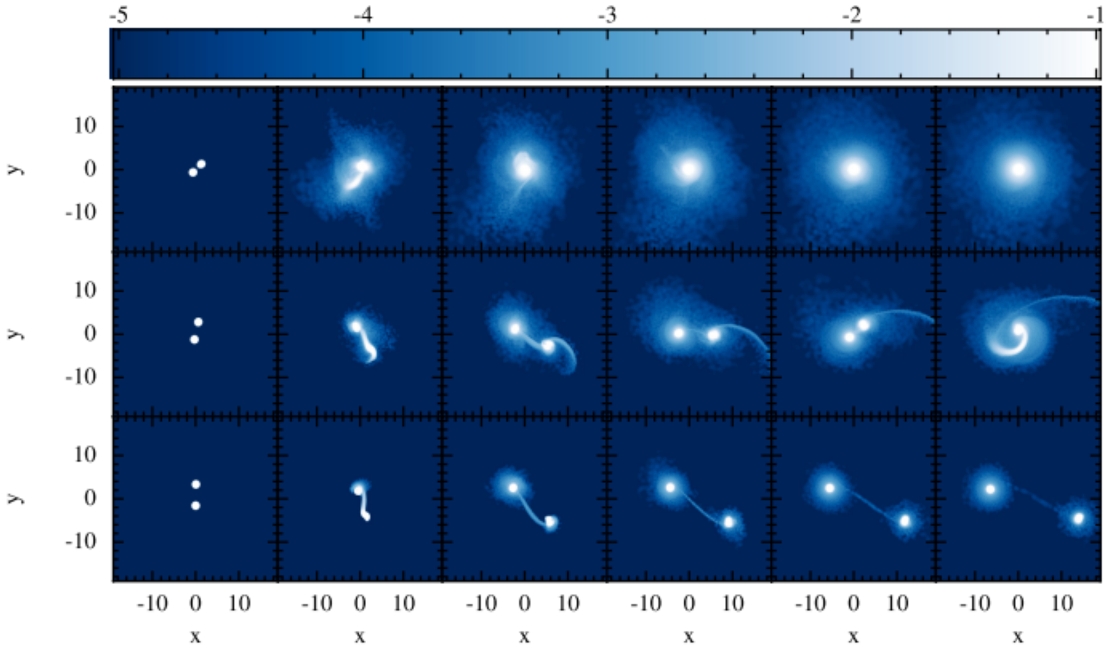


Eccentricity distribution induced by planet-planet scatterings (*Jurić & Tremaine 2008*)

Tidal effects in planet-planet scatterings

SPH simulation of close encounters between two gas giants (*JL+ 2021*):

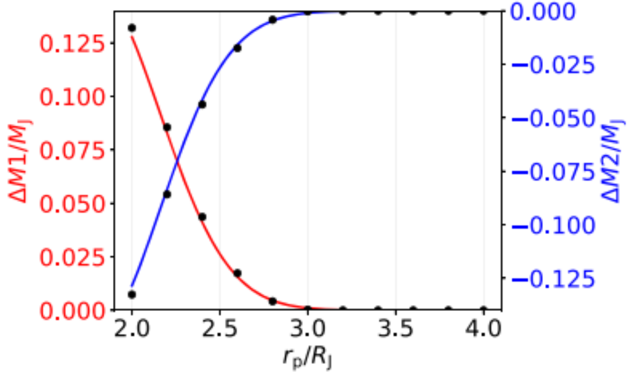
- colormap: log column density of gas
- column: snapshots at different times
- row: three different simulations

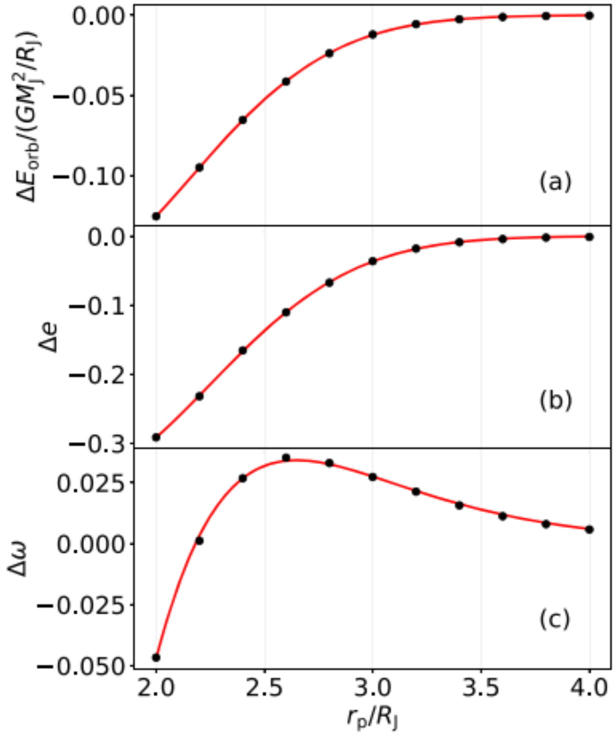


Tidal effects in planet-planet scatterings

SPH simulation of close encounters between two gas giants (*JL+ 2021*):

- mass transfer
- energy dissipation, circurlarization, and precenter shift of the relative orbits

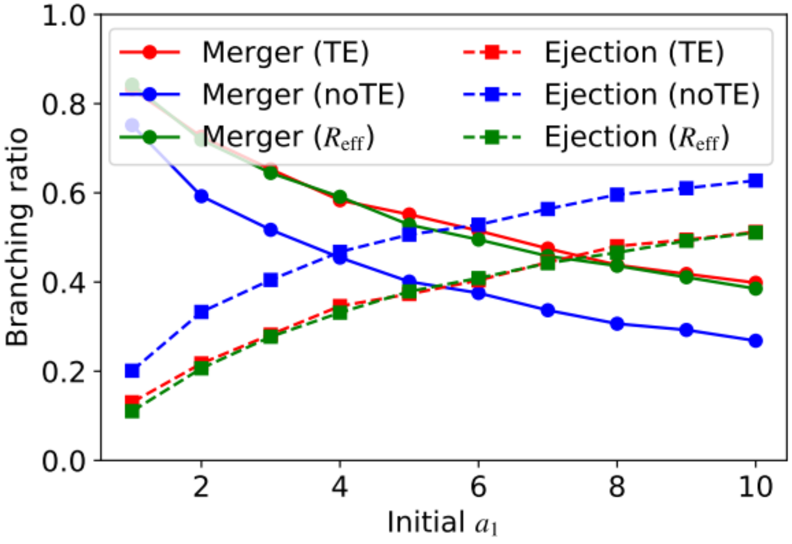




Planet-planet scatterings

- Planetary ejections:
  - Produce high-eccentricity planets
  - Excite planetary orbital inclination

- Planet-planet mergers:
  - Do not excite much eccentricities and inclinations

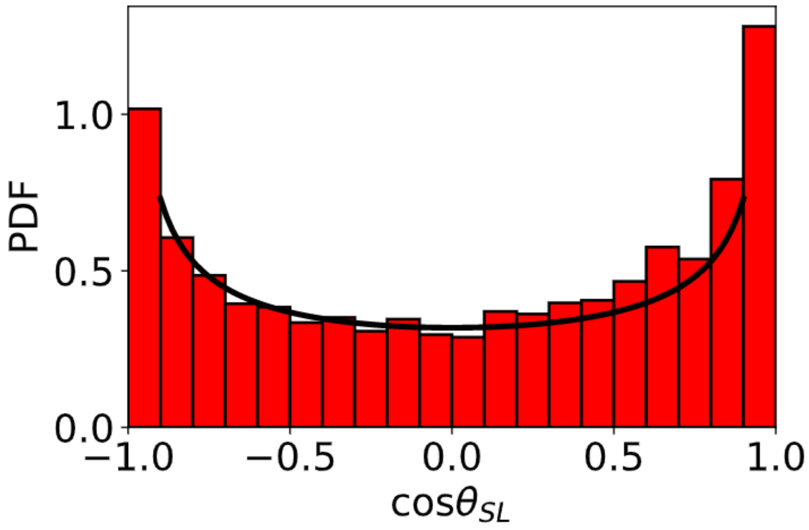


Tidal effects increase the probability of mergers and decrease the decrease the rate of ejections (*JL+ 2021*).

Planet-planet scatterings

- Planetary ejections:
  - Produce high-eccentricity planets
  - Excite planetary orbital inclinations

- Planet-planet mergers:
  - Do not excite much eccentricities and inclinations
  - **Do excite planets' spins and obliquities.**



Mergers can produce a population of rapidly spinning giant planets of a unique obliquity distribution (***JL*** *& Lai 2020*).

Part I Summary:

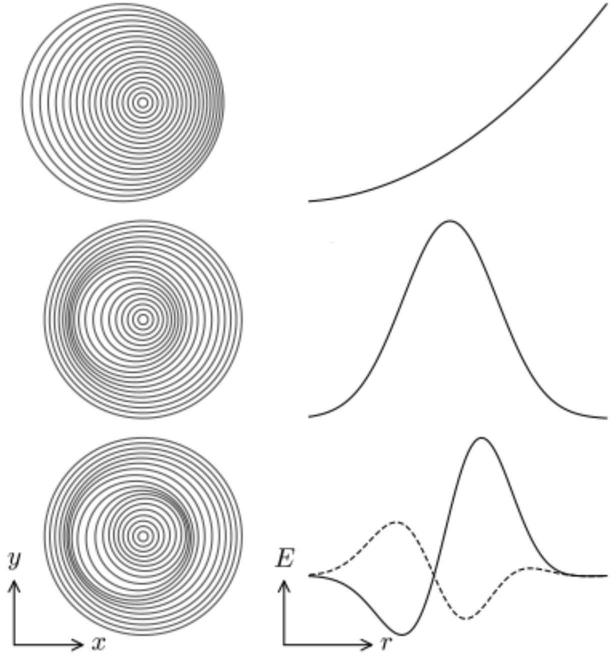
- Planet-planet scatterings is an important mechanism for producing high-eccentricity planets.
- Tidal effects can affect the efficiency of eccentricity excitation.
- Mergers of planets produce a unique distribution of planetary obliquity.

Part II: Theories of eccentric protoplanetary disks

Definition of disk eccentricity

E(r): complex eccentricity of a disk at each radius:

- $|E(r)|$: "orbital" eccentricity
- $\arg[E(r)]$: longitude of pericenter



an illustration of disk eccentricity
(*Lee+ 2019*)

Evolution equation and normal modes

- Evolution equation of disk eccentricity (*JL+ 2021*) :

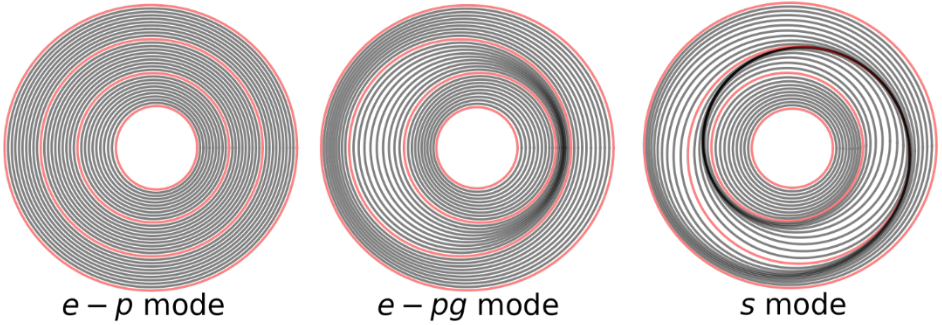
$$2r^3\Omega_{\rm K}\Sigma\frac{\partial E}{\partial t} = \left[-\frac{\beta}{i\beta+1}\mathcal{M}_{\rm adi} + \frac{i}{i\beta+1}\mathcal{M}_{\rm iso} + \mathcal{M}_{\rm sg} + \mathcal{M}_{\beta}\right] E$$

- Disk eccentric modes:

$$\partial E_{\rm m}/\partial t = i\omega_{\rm d,m}E_{\rm m}$$

Eccentric modes

$$\partial E_{\mathrm{m}}/\partial t = i\omega_{\mathrm{d,m}} E_{\mathrm{m}}$$



Disk eccentric modes: the complex eccentricity profiles that evolve coherently across their host disks. (*Lee+ 2019*)

disk eccentric modes:

- e-p mode: real and monotonic, pressure-dominated;
- e-pg mode: real, dominated by both pressure and disk self-gravity
- s mode: complex, self-gravity-dominated

A fiducial hydrosimulation

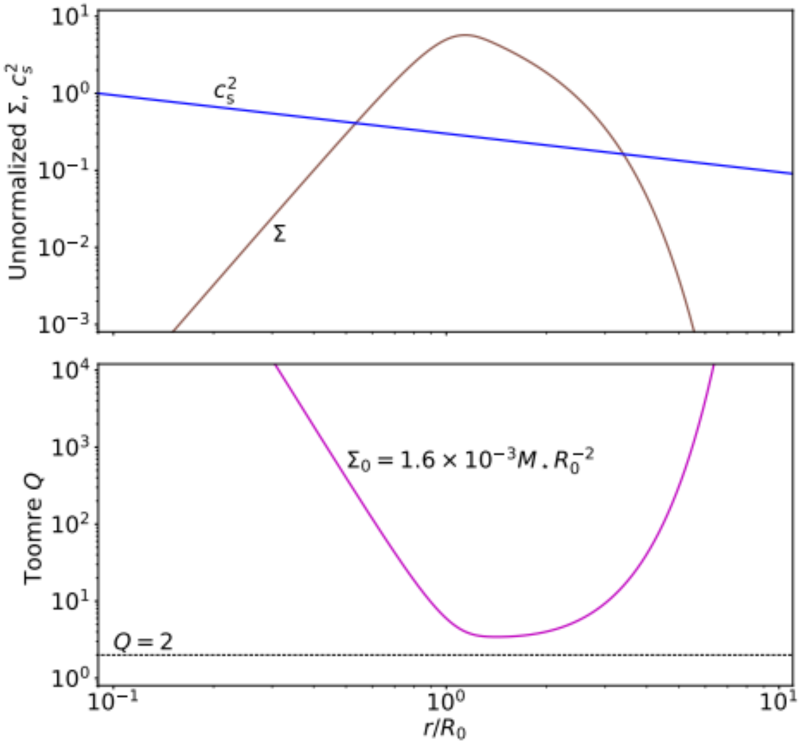
- Initial density:

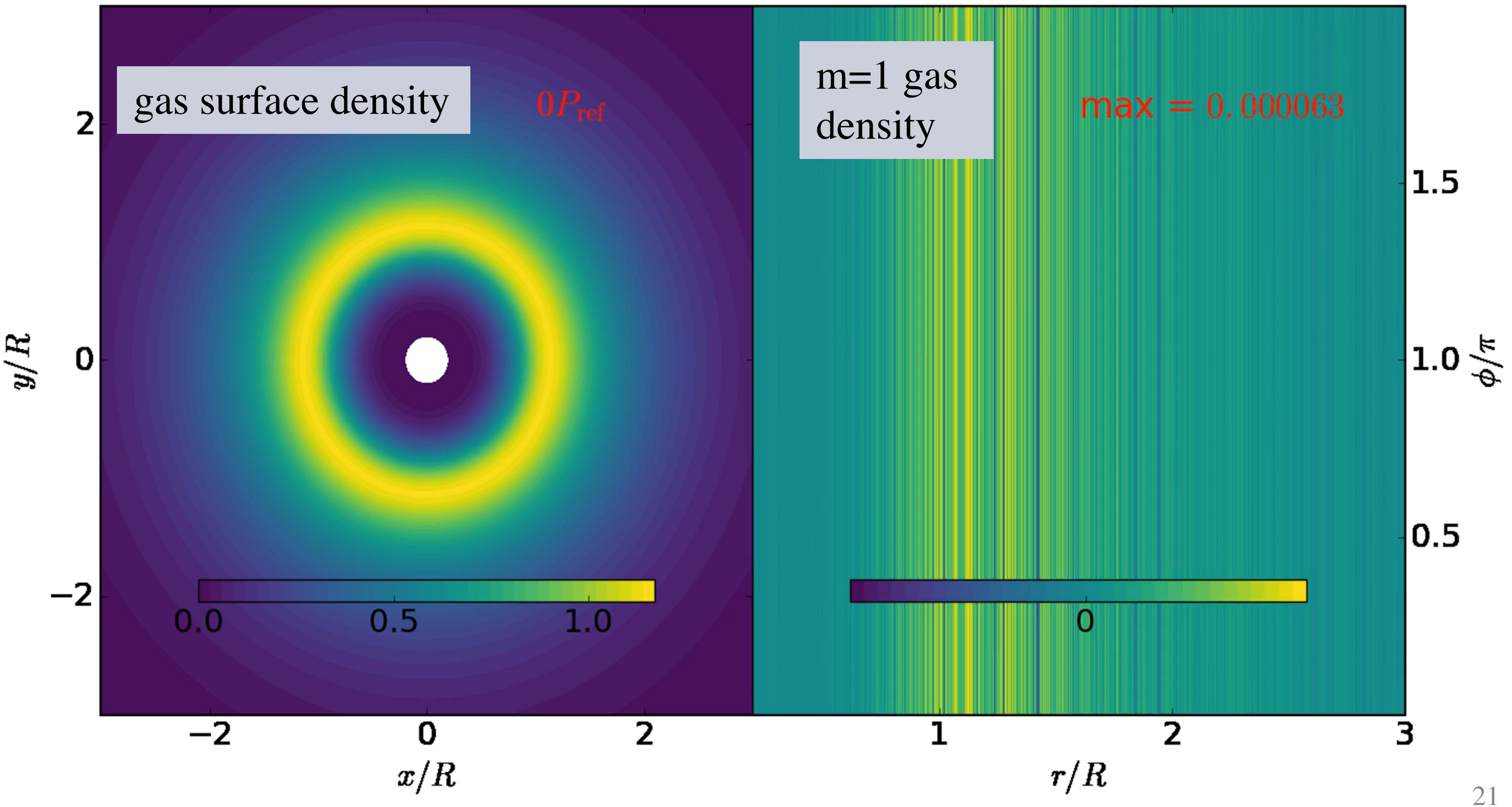
$$\Sigma(r) = 2.03\Sigma_0 \underbrace{\left(1 - e^{-(r/R_0)^6}\right)}_{\text{inner hole}} \overbrace{\left(\frac{R_0}{r}\right)}^{\text{power-law}} \underbrace{e^{-(r/(2R_0))^2}}_{\text{outer taper}}$$

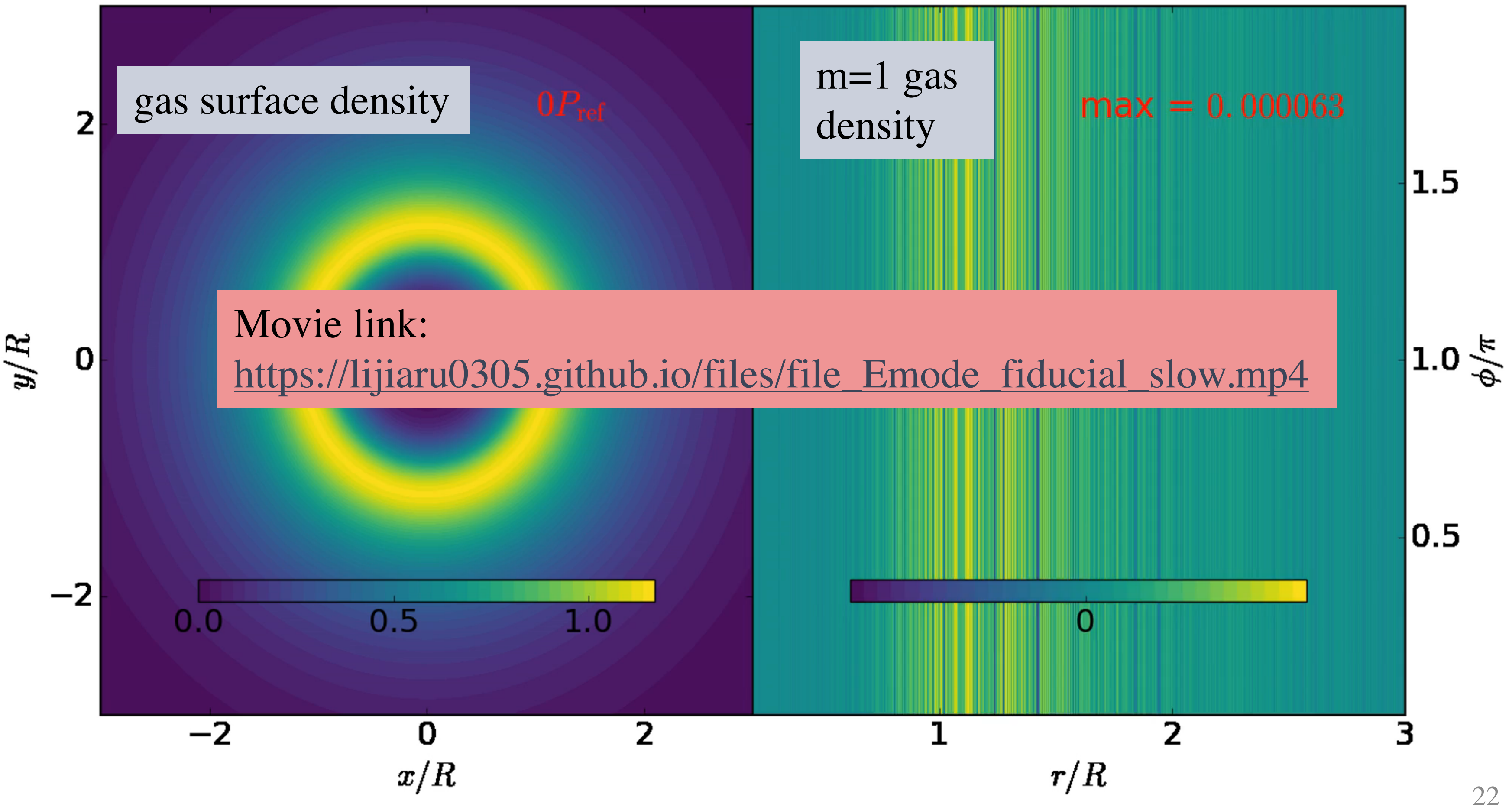
- Temperature:

$$c_s^2(r) = \gamma(k_b/\mu)T = c_0^2(r/R_0)^{-1/2}$$

with γ=1.5, c_0=0.03, and a cooling rate β=1e-6.

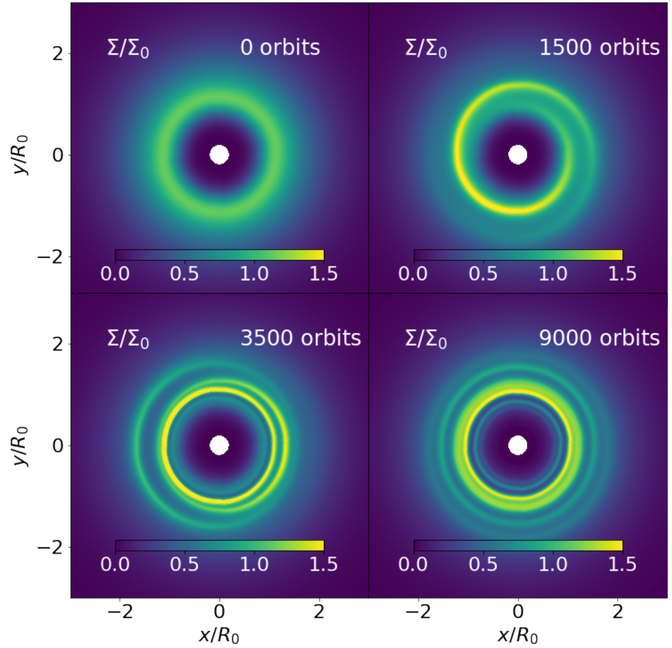



gas surface density
$0P_{\mathrm{ref}}$
m=1 gas density
max = 0.000063
2
0
−2
y/R
−2
0
2
x/R
0.0
0.5
1.0
1
2
3
r/R
1.5
1.0
0.5
φ/π
0

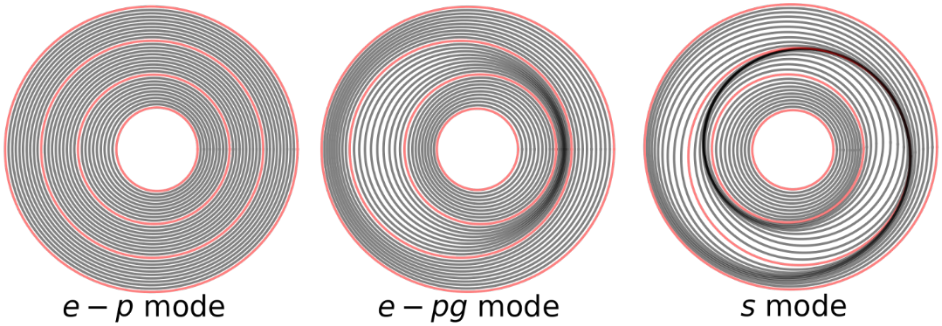


Movie link:
https://lijiaru0305.github.io/files/file_Emode_fiducial_slow.mp4

Spontaneous emergence of eccentric mode



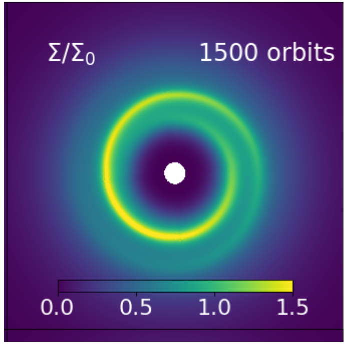
Ring and gap formations driven by the eccentric mode instability. (*JL+ 2021*)

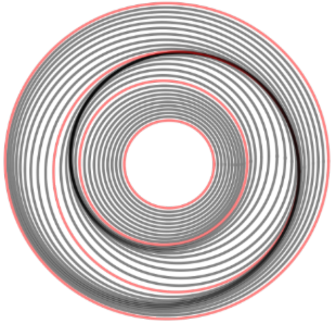


Eccentric mode instability (EMI):

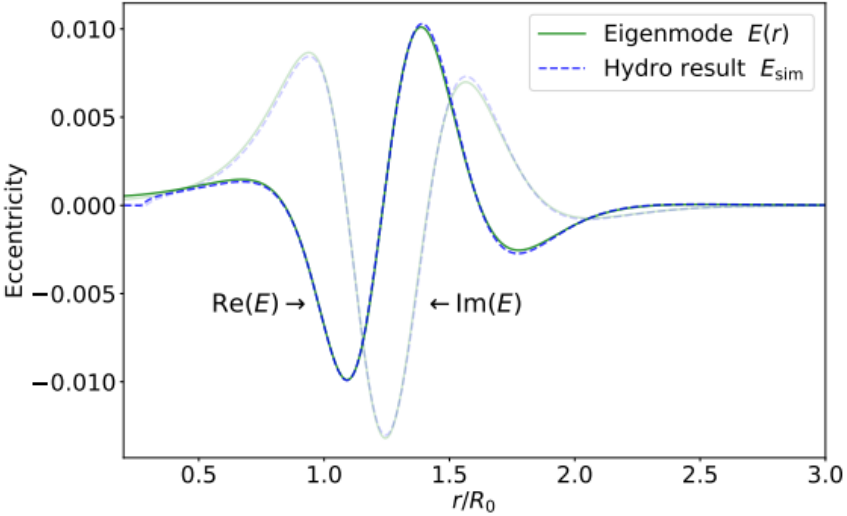
- the s mode grows exponentially by itself (*Lin 2015*);
- the disk transform into other e modes after saturation (*JL+ 2021*).

EMI: linear theory vs hydro simulation



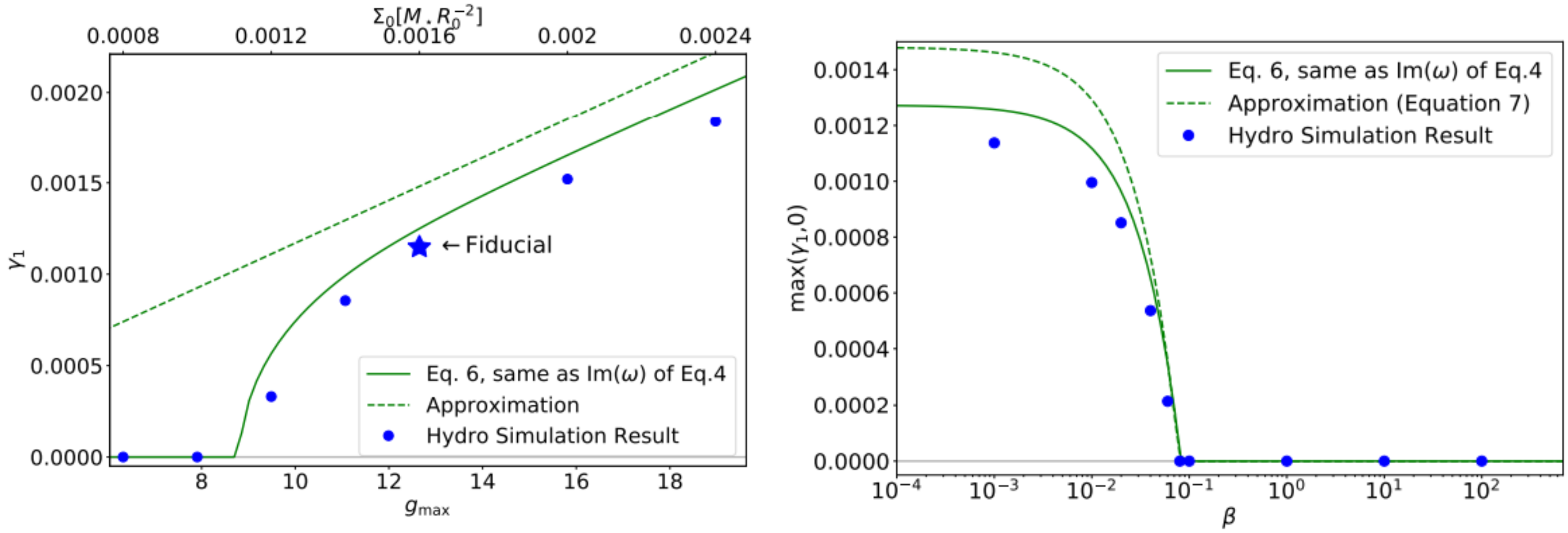


s mode



The linear mode matches the simulation result precisely.

EMI: growth rates and conditions.



Two conditions for EMI: - strong disk self-gravity - fast gas cooling

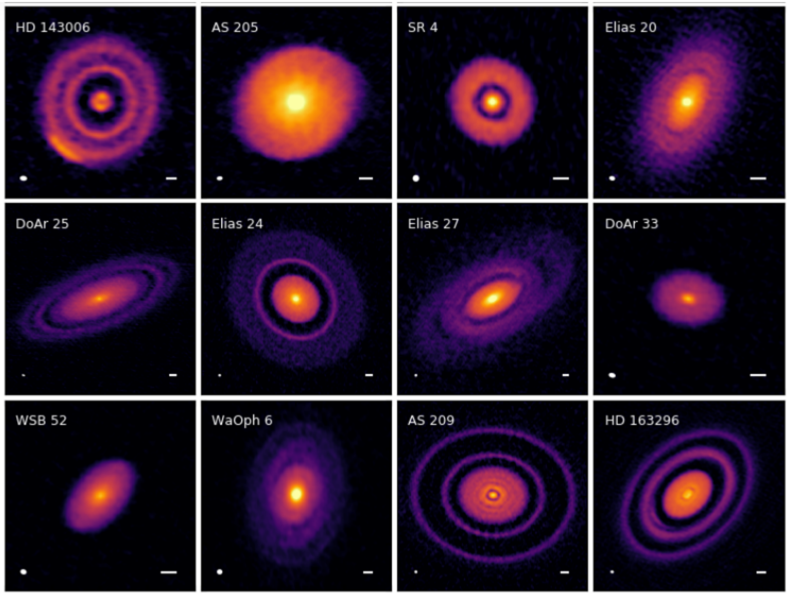
Part II Summary:

- Like planets, protoplanetary disks can also “store” eccentricities.
- Disks can have eccentric normal modes that allows disk eccentricities to precess/grow coherently.
- Under certain conditions, EMI allows a disk to become eccentric by itself.

Bonus: Rings in disks

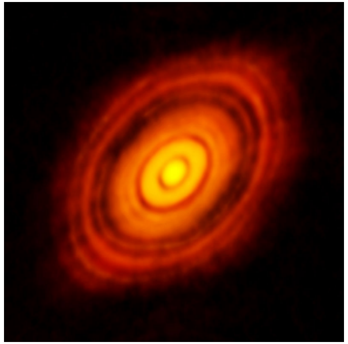
Protoplanetary disks:

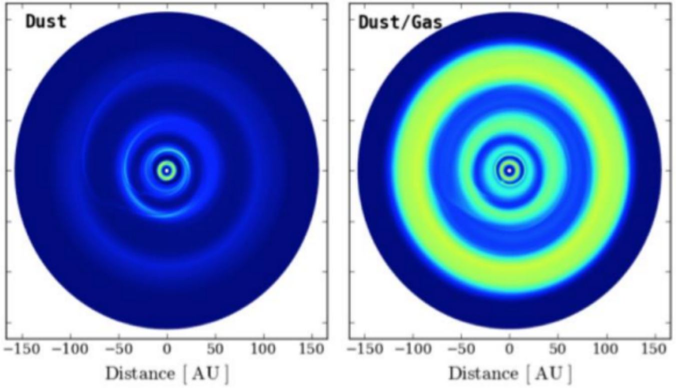
- commonly exhibit **substructures** (e.g., **rings and gaps**, inner cavities, vertices, spirals)



a gallery of 1.25mm continuum image for disks in DSHARP sample
(*Andrews et al., 2018*)

Conventional wisdom: planets





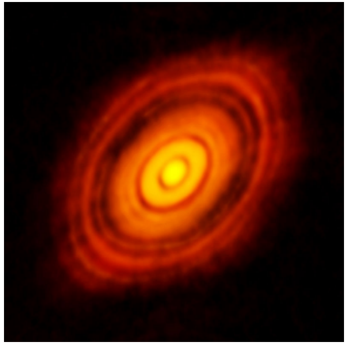
Top: ALMA image of HL Tau;
Bottom: simulation (*Jin+ 2016* @ LANL)
0.35, 0.17, 0.26 M_J @ 13.1, 33.0, 68.6 AU

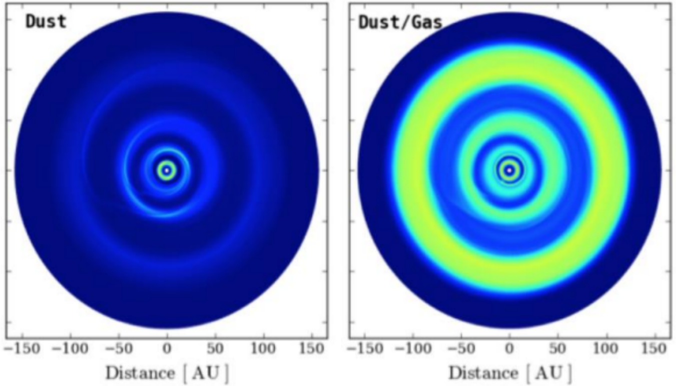
Mechanism:

- Planets orbiting around the star while being **embedded** in the disk.
- **Each planet carves a gap** around its orbit.

Issues:

- Disk rings have very **large radii**.
- Rings are found in **young disks**, too.





Top: ALMA image of HL Tau;
Bottom: simulation (*Jin+ 2016* @ LANL)
0.35, 0.17, 0.26 M_J @ 13.1, 33.0, 68.6 AU

Conventional wisdom: planets

Mechanism:

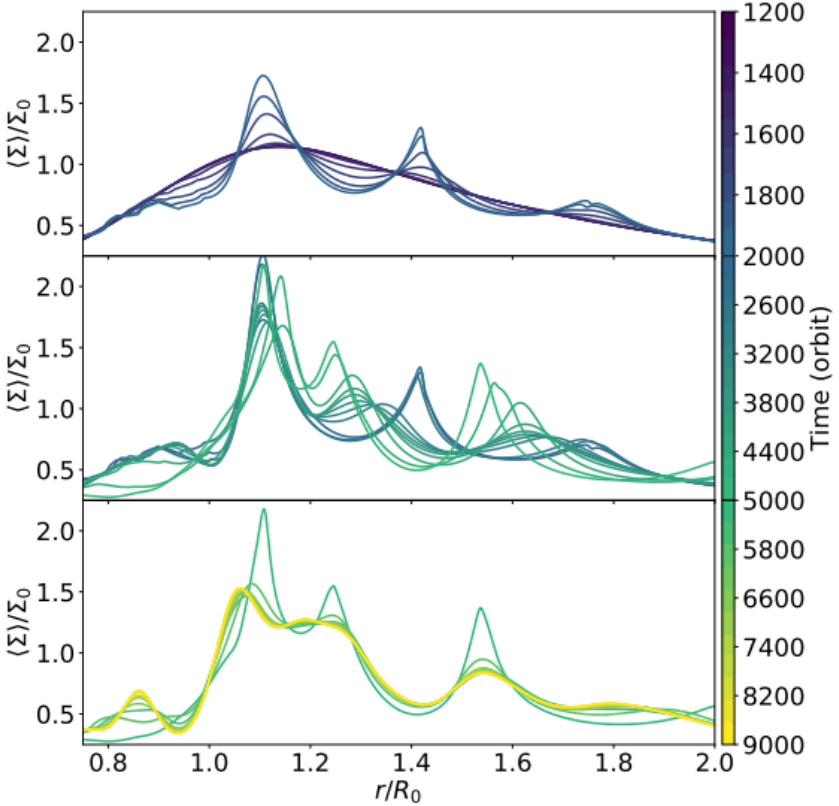
- Planets orbiting around the star while being **embedded** in the disk.
- **Each planet carves a gap** around its orbit.

We have showed that **eccentric mode instability (EMI)** can generate these **rings** without the help of planets

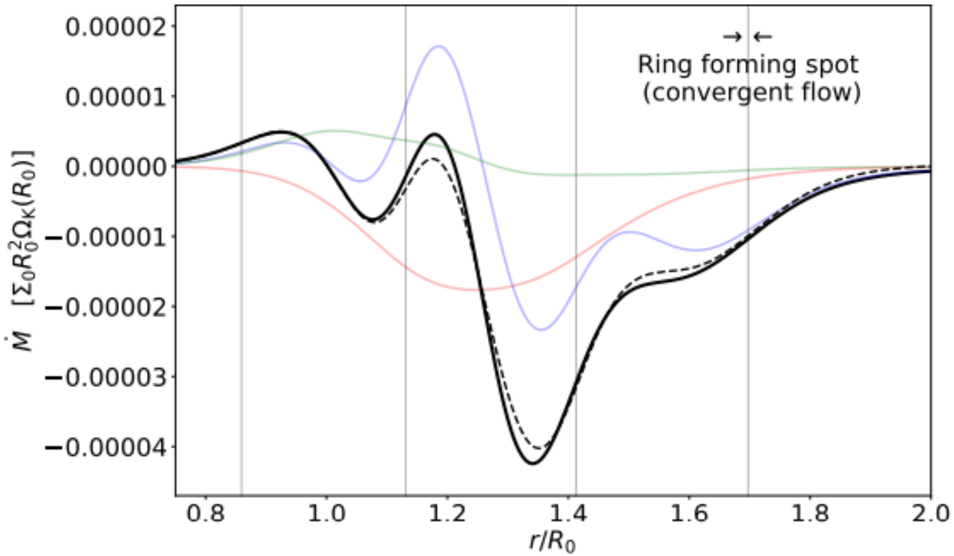
Ring formation via EMI: hydro simulation

Time evolution of the azimuthally averaged density profile:

- Multiple rings are formed during the EMI exponential growth stage (top panel).
- The follow-up evolution relax the position and amplitude of the rings (middle and lower panel).

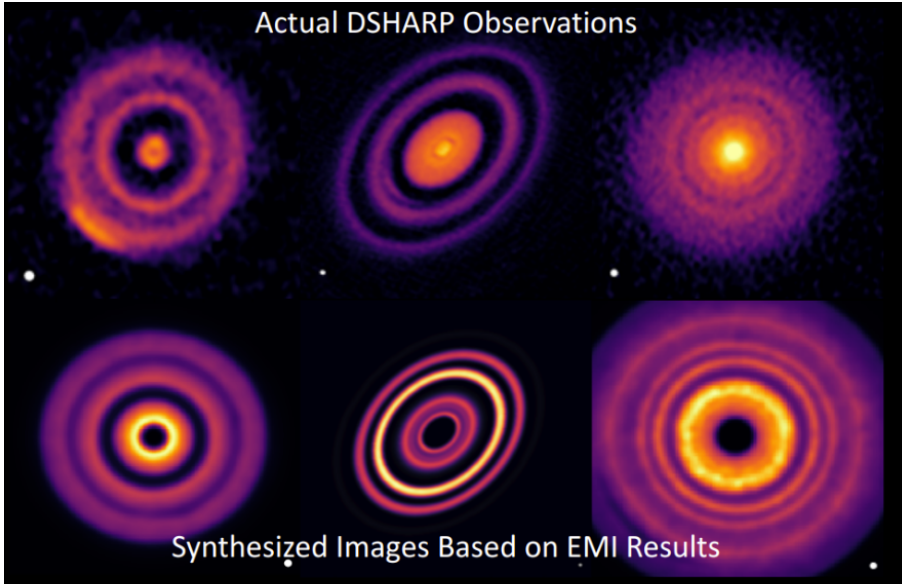


Ring formation via EMI: linear theory prediction



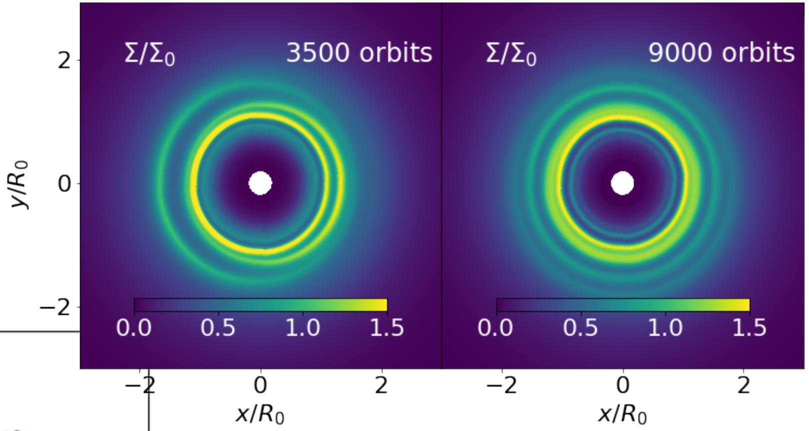
We can also use first order non-linear coupling to predict the location of the rings (black solid: theory, black dashed: simulation)

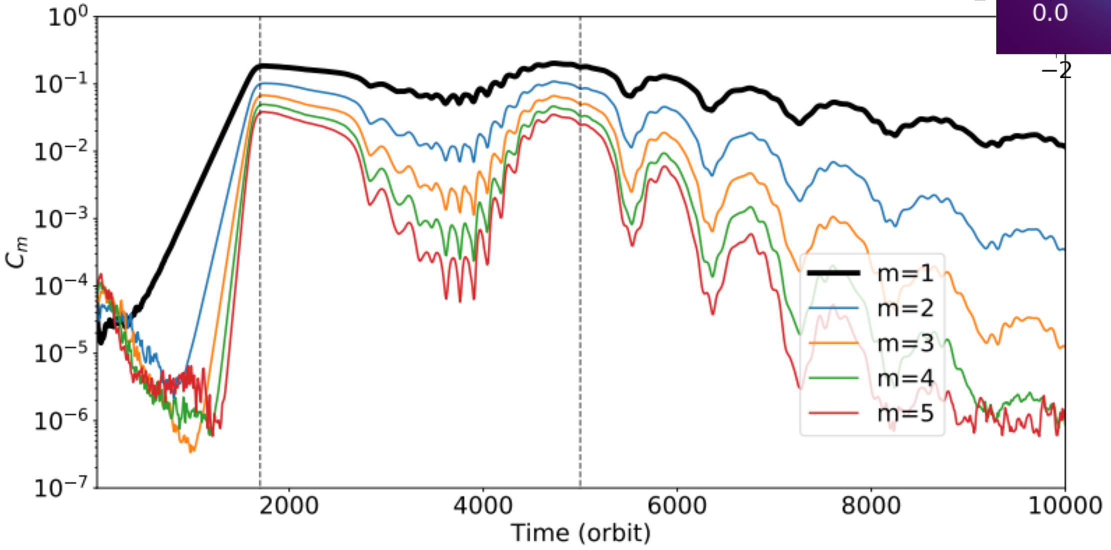
EMI rings vs DSHARP rings (*JL+, in prep*)



- Start with a ringless density profile.
- Evolve the disks with hydro simulations
- **Add dust** at different stages and continue evolving with the hydro code.
- Run the **Monte-Carlo radiative transfer simulation RADMC-3D** to get **synthetic images**.

Final disk morphology



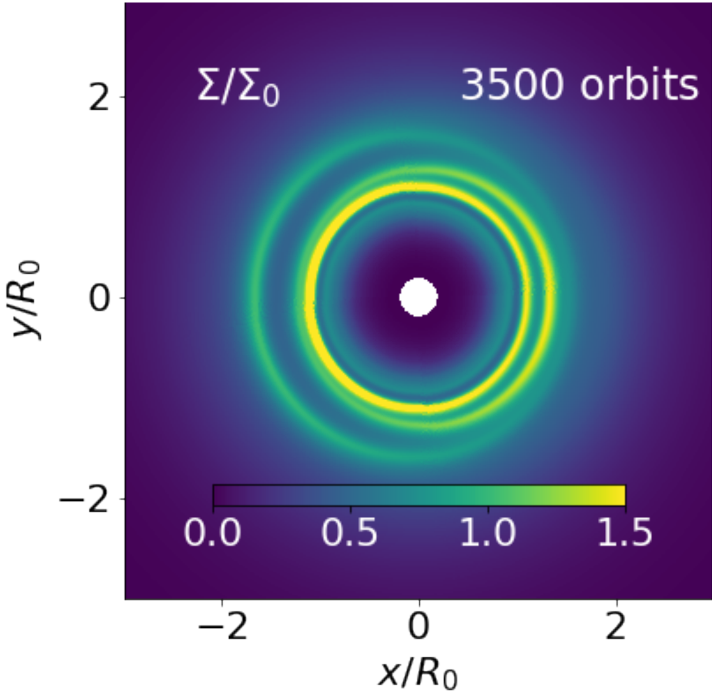


Long-term evolution steps:

1. EMI (s mode) saturates.
2. Disk maintains its eccentricity in a long-lived e mode.
3. If there is damping, rings because circular.

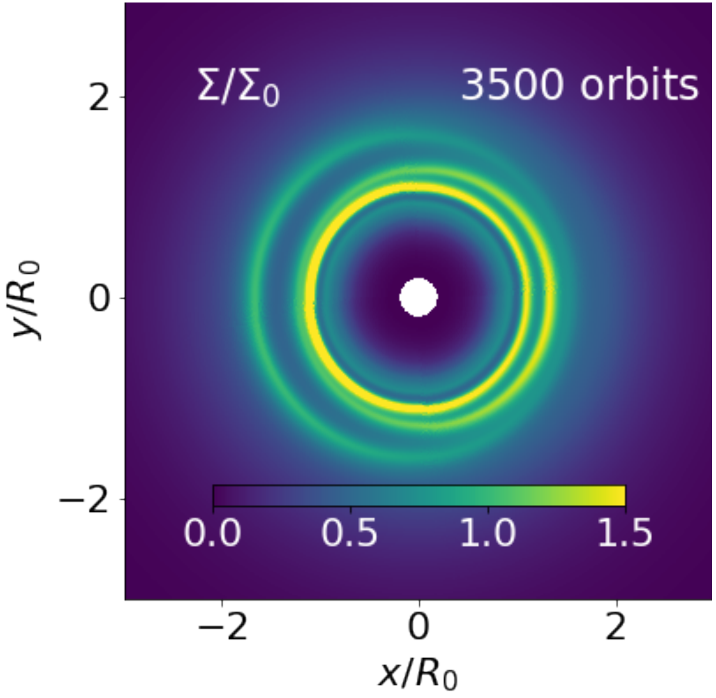
Part III: resonant excitation of planetary eccentricity due to a dispersing eccentric disk

A new mechanism to generate planet eccentricity



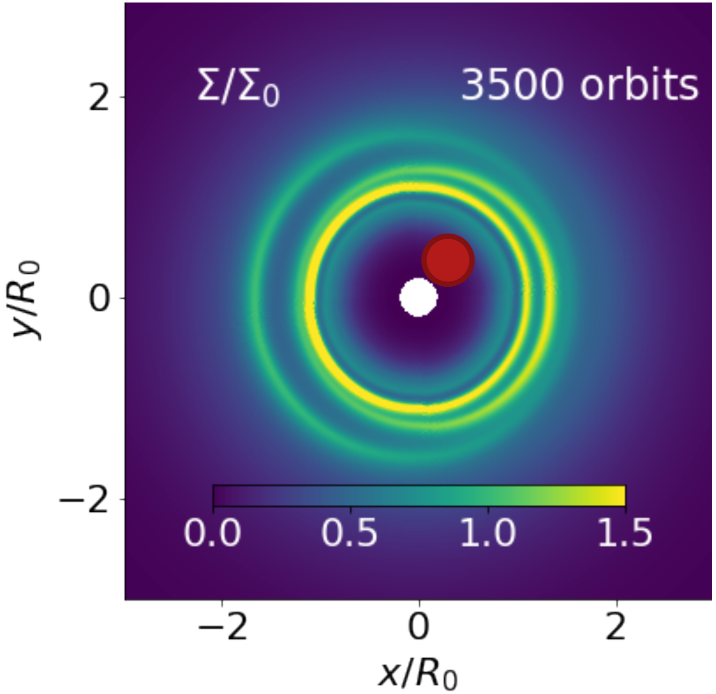
- Disk has a small initial eccentricity. (Eccentric mode)

A new mechanism to generate planet eccentricity



- Disk has a small initial eccentricity. (Eccentric mode)
- The disk loses mass with time.

A new mechanism to generate planet eccentricity



- Disk has a small initial eccentricity. (Eccentric mode)
- The disk loses mass with time.
- Planet inside the inner cavity of the disk.

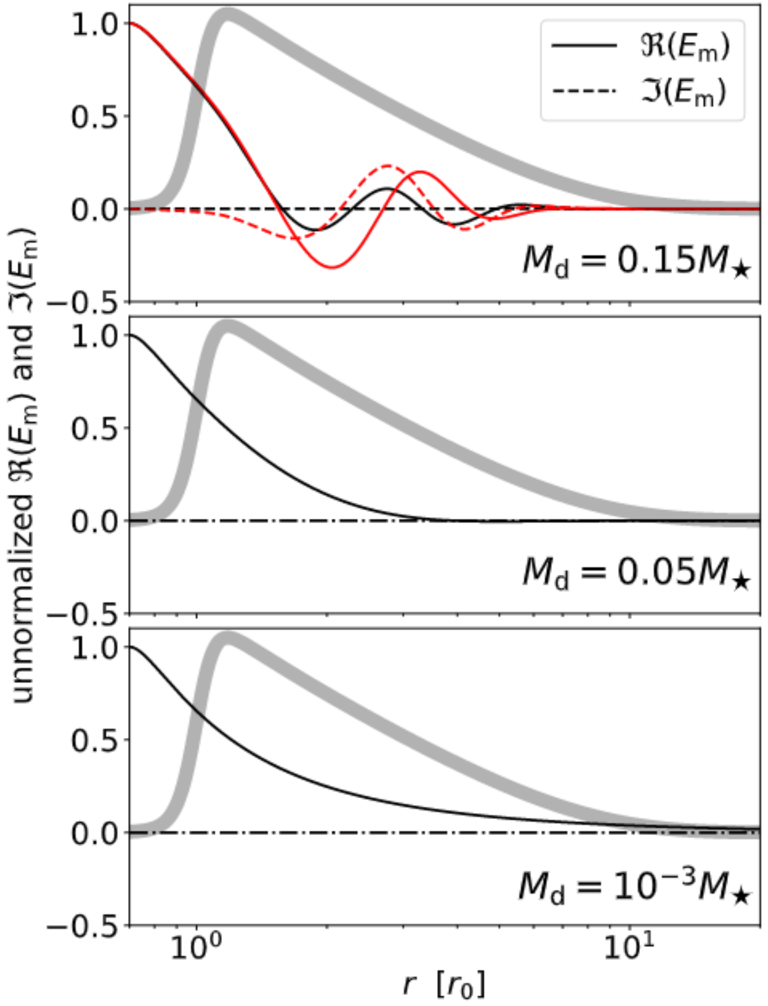
Eccentric mode in a dispersing disk

- Evolution equation:

$$2r^3\Omega_{\rm K}\Sigma\frac{\partial E}{\partial t} = \left(-\frac{\beta}{i\beta+1}\mathcal{M}_{\rm adi} + \frac{i}{i\beta+1}\mathcal{M}_{\rm iso} + \mathcal{M}_{\rm sg} + \mathcal{M}_{\beta}\right)E,$$

- Eccentric modes:

$$\partial E_{\rm m}/\partial t = i\omega_{\rm d,m}E_{\rm m}$$



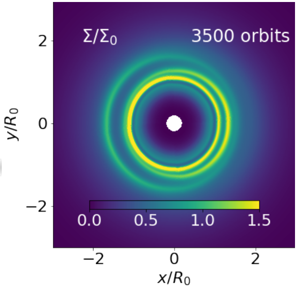
Eccentric mode in a dispersing disk

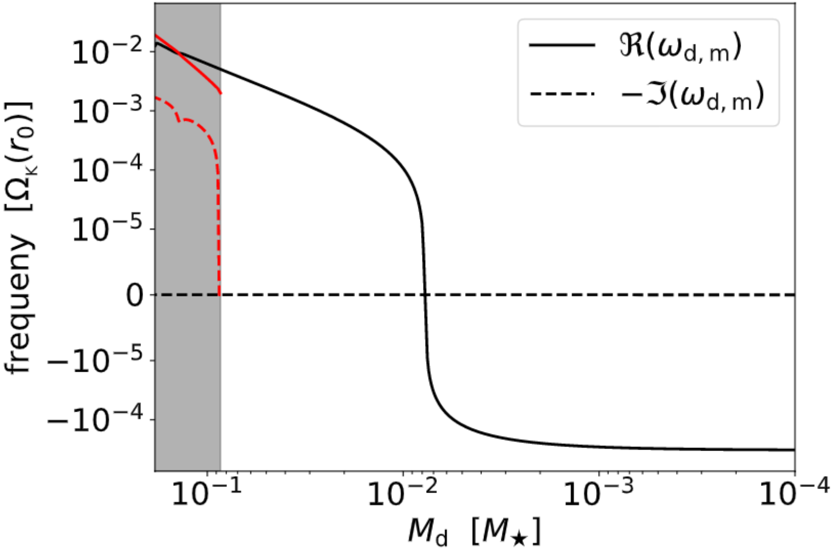
- Evolution equation:

$$2r^3\Omega_{\rm K}\Sigma\frac{\partial E}{\partial t} = \left(-\frac{\beta}{i\beta+1}\mathcal{M}_{\rm adi} + \frac{i}{i\beta+1}\mathcal{M}_{\rm iso} + \mathcal{M}_{\rm sg} + \mathcal{M}_{\beta}\right)E,$$

- Eccentric modes:

$$\partial E_{\rm m}/\partial t = i\omega_{\rm d,m}E_{\rm m}$$





Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (Teyssandier & Lai 2019)

$$\begin{aligned}\frac{dE_{\rm d}}{dt} &= i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p} \\ \frac{dE_{\rm p}}{dt} &= -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}\end{aligned}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r, t; M_d)$ has the same "shape" as the e mode of the disk

$$E(r, t; M_{\rm d}) = E_{\rm m}(r; M_{\rm d}) E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (Teyssandier & Lai 2019)

e mode frequency

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_\mathrm{d}) = E_\mathrm{m}(r;M_\mathrm{d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (Teyssandier & Lai 2019)

mutually induced precession rate

$$\frac{dE_\mathrm{d}}{dt} = i(\omega_\mathrm{d,m} + \omega_\mathrm{d,p})E_\mathrm{d} - i\nu_\mathrm{d,p}E_\mathrm{p}$$

$$\frac{dE_\mathrm{p}}{dt} = -i\nu_\mathrm{p,d}E_\mathrm{d} + i\omega_\mathrm{p,d}E_\mathrm{p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (Teyssandier & Lai 2019)

eccentricity coupling rate

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$

$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

Planet-disk interaction model

- Assume that the disk's coherent eccentricity $E(r,t;M_d)$ has the same "shape" as the e mode of the disk

$$E(r,t;M_{\rm d}) = E_{\rm m}(r;M_{\rm d})E_d(t)$$

- We can write down the eccentricity interaction equation for a planet and 'rigid' disk as (Teyssandier & Lai 2019)

$$\frac{dE_{\rm d}}{dt} = i(\omega_{\rm d,m} + \omega_{\rm d,p})E_{\rm d} - i\nu_{\rm d,p}E_{\rm p}$$
$$\frac{dE_{\rm p}}{dt} = -i\nu_{\rm p,d}E_{\rm d} + i\omega_{\rm p,d}E_{\rm p}$$

$$\omega_{\rm d,p} = \frac{1}{J_{\rm d}}\int GM_{\rm p}\Sigma K_1(r,a_{\rm p})|E_{\rm m}|^2 2\pi r dr$$
$$\omega_{\rm p,d} = \frac{1}{J_{\rm p}}\int GM_{\rm p}\Sigma K_1(r,a_{\rm p})2\pi r dr$$
$$\nu_{\rm d,p} = \frac{1}{J_{\rm d}}\int GM_{\rm p}\Sigma K_2(r,a_{\rm p})E_{\rm m}2\pi r dr$$
$$\nu_{\rm p,d} = \frac{1}{J_{\rm p}}\int GM_{\rm p}\Sigma K_2(r,a_{\rm p})E_{\rm m}2\pi r dr$$

Numerical examples

Result of the long-term time evolution in the fiducial system:

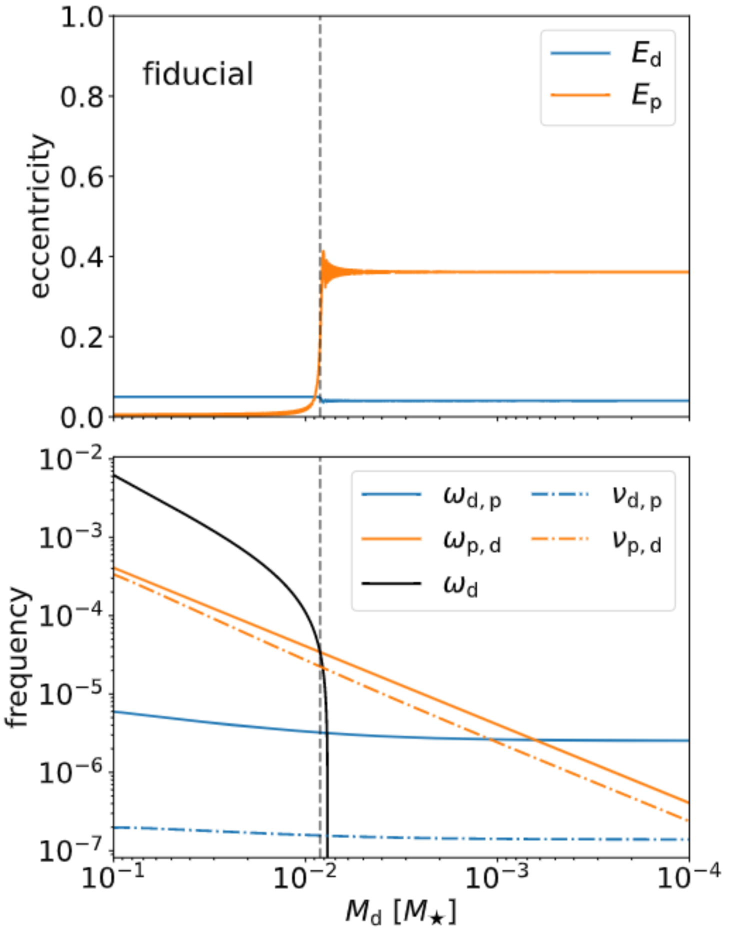
- Planet

$$a_{\rm p} = 0.2 r_0, \ M_{\rm p} = 3 \times 10^{-4} M_\odot$$

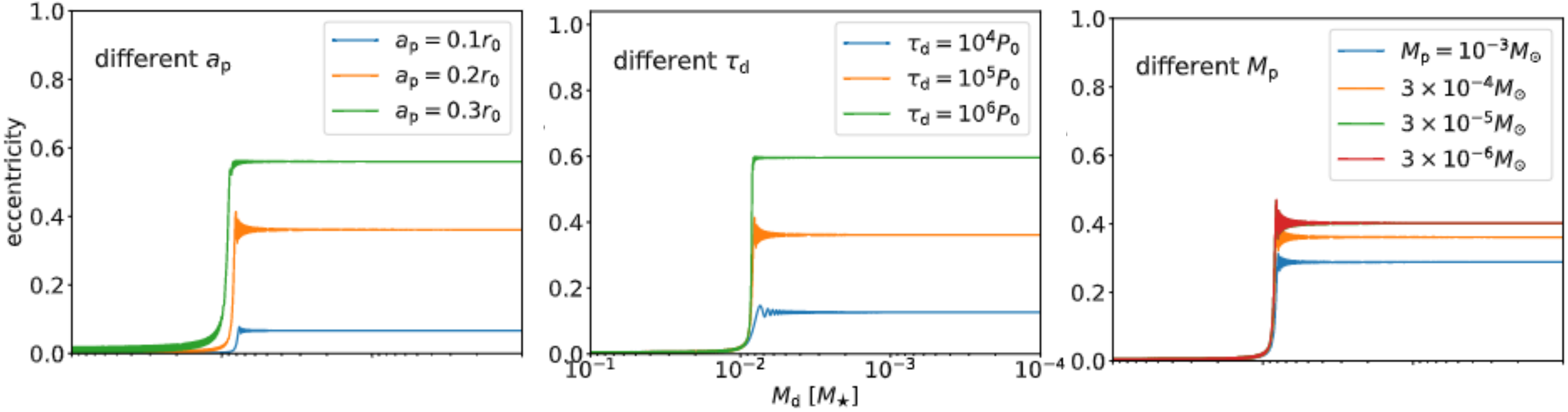
- Disk

$$M_{\rm d}(t) = \frac{M_{\rm d,0}}{1 + t/\tau_{\rm d}}, \ \tau_{\rm d} = 10^5 P_0$$

$$M_{\rm d,0} = 0.1 M_\star = 0.1 M_\odot$$



Numerical examples: parameter study



Analysis

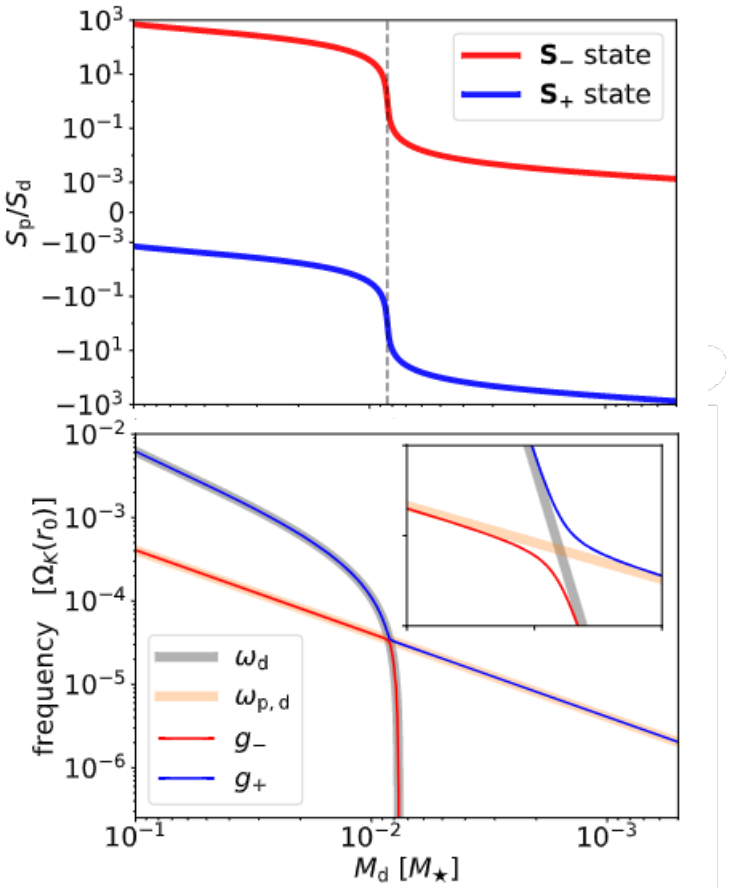
- It is useful to consider two different variables:

$$S_{\rm d} = \left(\frac{J_{\rm d}}{2}\right)^{1/2} E_{\rm d} \qquad S_{\rm p} = \left(\frac{J_{\rm p}}{2}\right)^{1/2} E_{\rm p}$$

- The two variables evolve as

$$\frac{d}{dt}\begin{pmatrix} S_{\rm d} \\ S_{\rm p} \end{pmatrix} = i \begin{pmatrix} \omega + \Delta\omega + i\frac{1}{2\tau_{\rm J}} & -\nu \\ -\nu & \omega - \Delta\omega \end{pmatrix} \begin{pmatrix} S_{\rm d} \\ S_{\rm p} \end{pmatrix}$$

- We can find their eigenstates.



Analysis

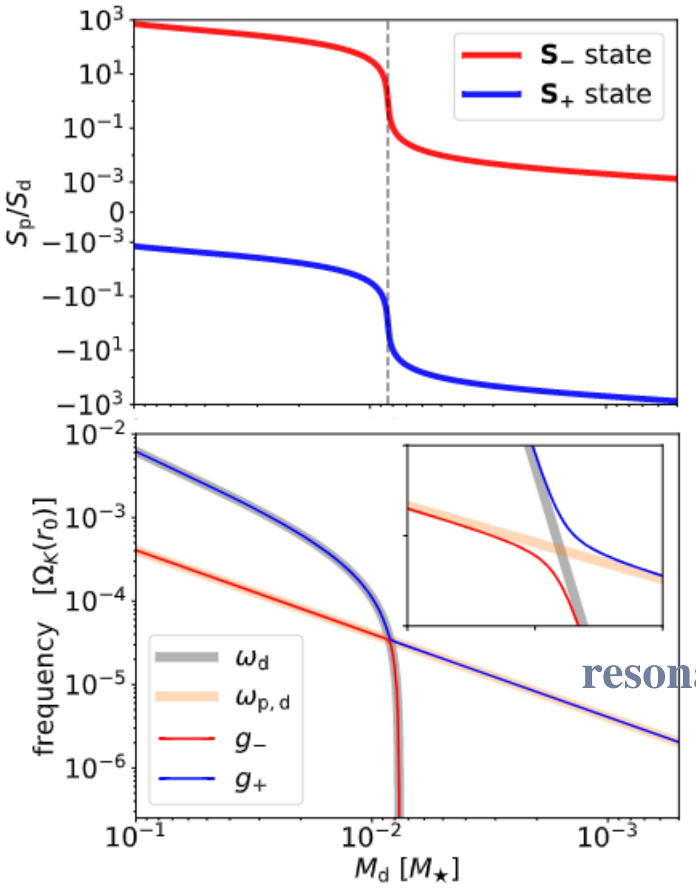
- It is useful to consider two different variables:

$$S_{\mathrm{d}} = \left(\frac{J_{\mathrm{d}}}{2}\right)^{1/2} E_{\mathrm{d}} \qquad S_{\mathrm{p}} = \left(\frac{J_{\mathrm{p}}}{2}\right)^{1/2} E_{\mathrm{p}}$$

- The two variables evolve as

$$\frac{d}{dt}\begin{pmatrix} S_{\mathrm{d}} \\ S_{\mathrm{p}} \end{pmatrix} = i \begin{pmatrix} \omega + \Delta\omega + i\frac{1}{2\tau_{\mathrm{J}}} & -\nu \\ -\nu & \omega - \Delta\omega \end{pmatrix} \begin{pmatrix} S_{\mathrm{d}} \\ S_{\mathrm{p}} \end{pmatrix}$$

- We can find their eigenstates.



resonance

Analysis

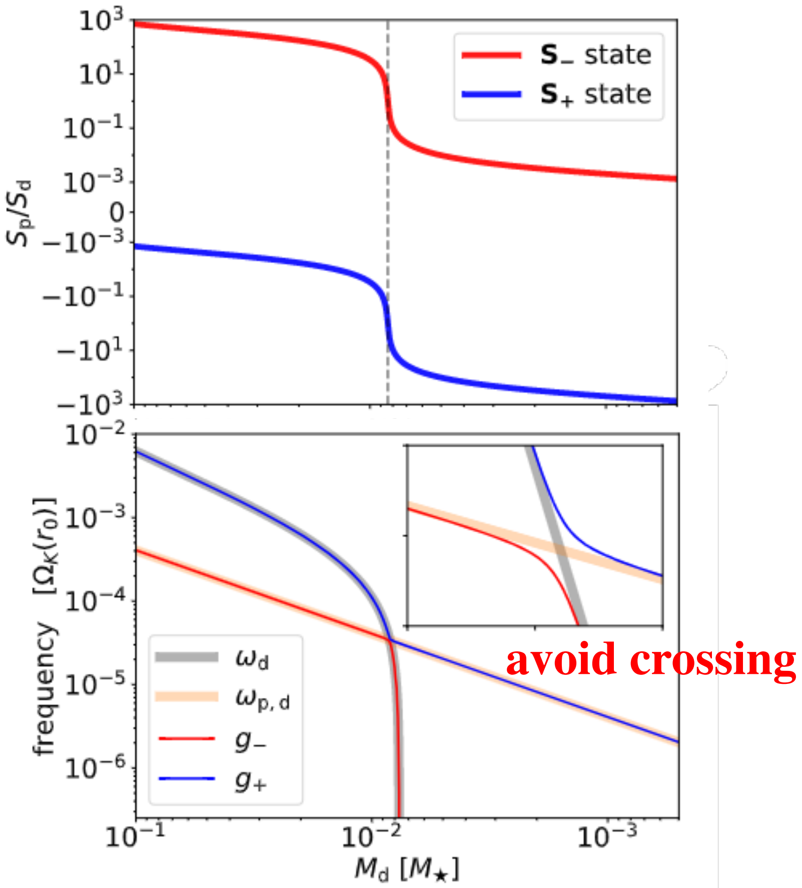
- It is useful to consider two different variables:

$$S_\mathrm{d} = \left(\frac{J_\mathrm{d}}{2}\right)^{1/2} E_\mathrm{d} \qquad S_\mathrm{p} = \left(\frac{J_\mathrm{p}}{2}\right)^{1/2} E_\mathrm{p}$$

- The two variables evolve as

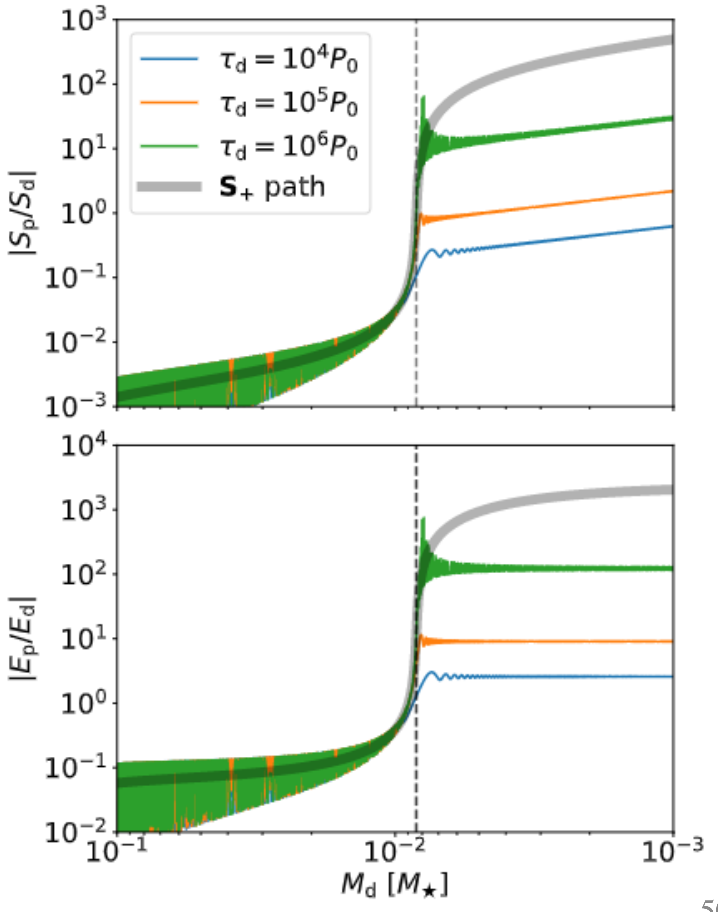
$$\frac{d}{dt}\begin{pmatrix} S_\mathrm{d} \\ S_\mathrm{p} \end{pmatrix} = i \begin{pmatrix} \omega + \Delta\omega + i\frac{1}{2\tau_\mathrm{J}} & -\nu \\ -\nu & \omega - \Delta\omega \end{pmatrix} \begin{pmatrix} S_\mathrm{d} \\ S_\mathrm{p} \end{pmatrix}$$

- We can find their eigenstates.



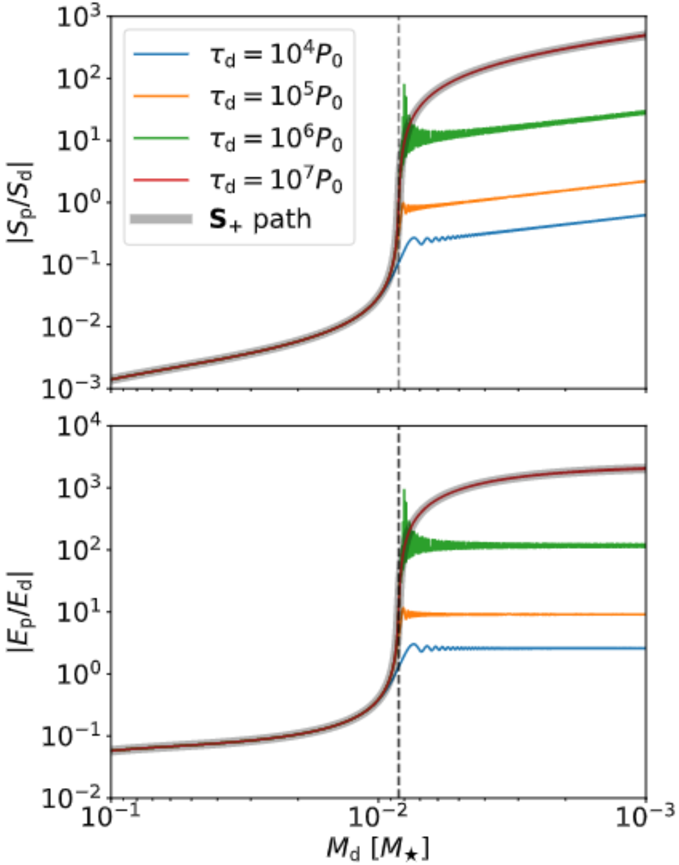
Analysis

- The time integrations start nearly in the S+ state.
- If the disk loses mass with time slowly, the eigenstate also evolves slowly.
- By adiabatic theorem, the eccentricities of the planet and the disk evolves accordingly to remain in the eigenstate.
- The eigenstate path naturally guide the system to a high-planet-eccentricity configuration.



Analysis

- We can also start exactly on the eigen-path and try larger disk dispersal timescale…



Summary of the new mechanism

- EMI allows massive disks to rapidly becomes eccentric.
- The long-term outcome of EMI is a disk with either multiple rings or long-lived eccentricity.
- A disk gradually loses mass while preserving a long-lived, precessing eccentricity can excite the eccentricity of a planet (up to 0.6).
- The resonant excitation of planetary eccentricity by an eigenmode analysis.