Steady-State Warp in Protoplanetary Disks:
From Nonlinear Theory to Breaking

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Abstract

Recent observations have found an increasing amount of evidence that many protoplanetary disks are warped or broken.

Using 3D hydrodynamic simulations, linear and nonlinear theories, we show that:

(1) Protoplanetary disks can maintain long-lived steady warps, whose properties are well-described by nonlinear theories.
(2) Disk breaking can occur inside strong warps, where the internal torques fully saturate.

Our results provide insights into how disks bend and sustain coherent warps, and how they break.

Basic Theory

Accretion disks are not always planar. Let *β(r)* be the orbital inclination of the gas at each disk radius:

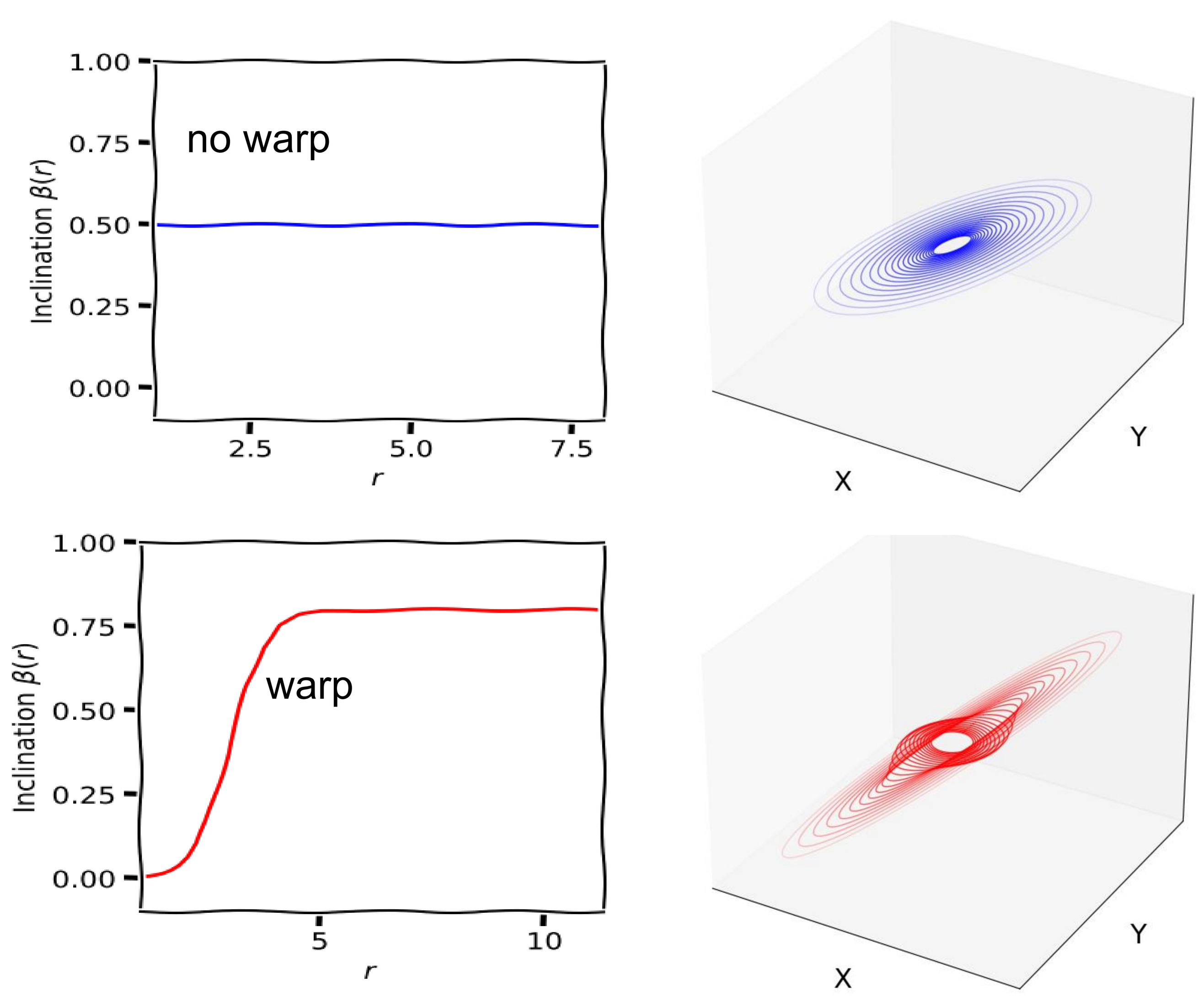


Figure 1. Sketches of a flat (top) and a warped disk (bottom).

Warp $|\psi|$ is formally defined as the radial derivative of the disk inclination: $|\psi| = |r\partial_r\beta|$
(i) Misaligned disk annuli exert *internal torques* on each other.
(ii) Internal torque drives changes of *β(r)*.

Hydrodynamic Simulation

To focus on disk hydrodynamics, while remaining agnostic about what external object forces the warp, we:

- fix the disk inclination angles $\beta_{in} = 0$ and vary β_{out}
- use a partial density gap to localize the warp

We simulate the 3D hydrodynamic evolution for our disk model using Athena++ (Stone *et al.* 2020).

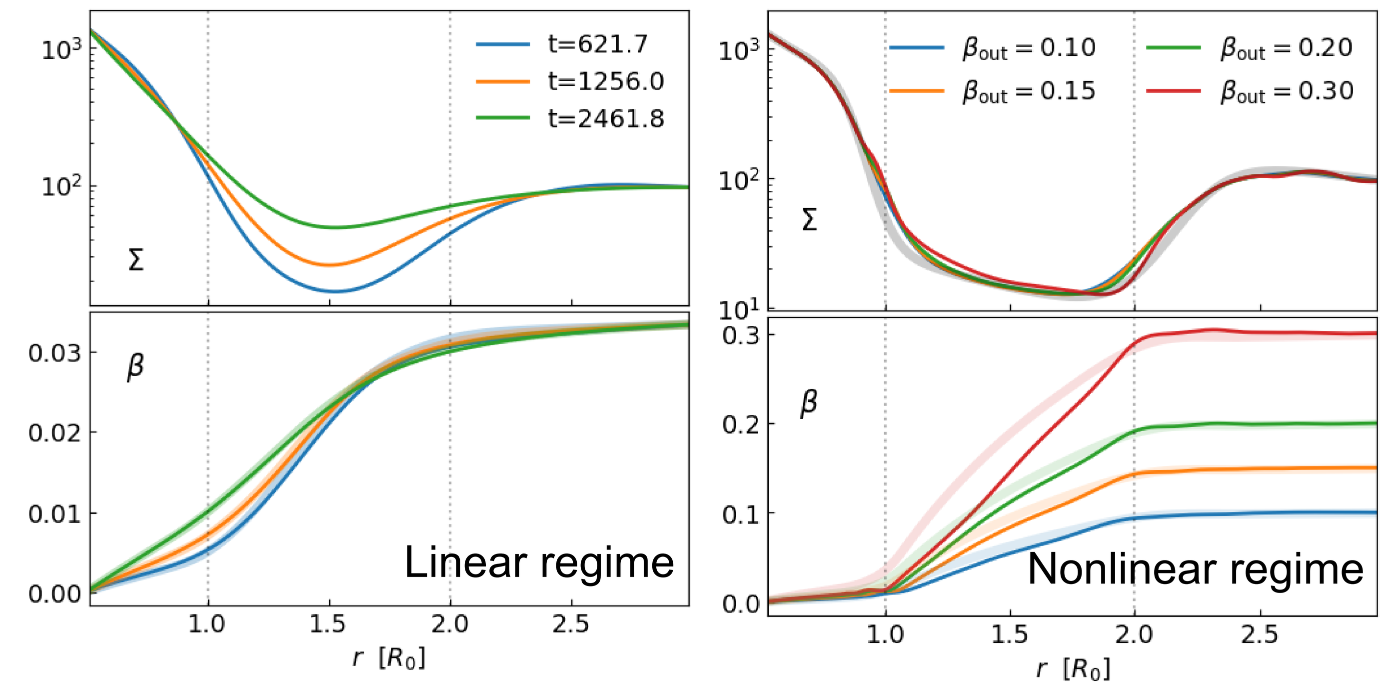


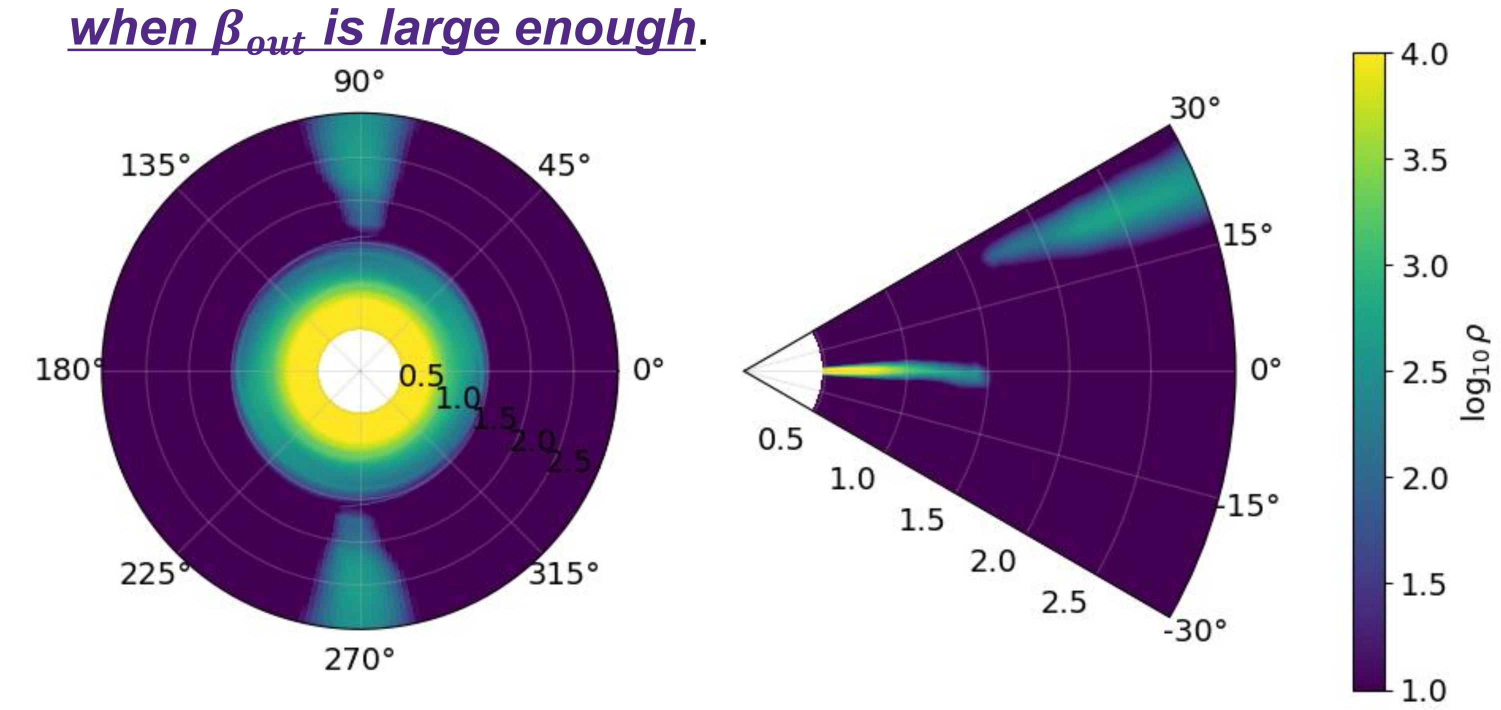
Figure 2. Simulation results for small $\beta_{out} = 0.033 = 2^o$ (left) and various larger β_{out} values. The warps have reached steady states.

- **Solid:** measured disk profiles from simulations
- **Faint:** linear theory predictions (e.g., Lubow & Ogilvie 2000)

*Simulation and the linear theory agree well for the **small β_{out}**, but deviate at **larger values**.*

Breaking

We find that the disk can break *into two misaligned flat plates **when β_{out} is large enough***.



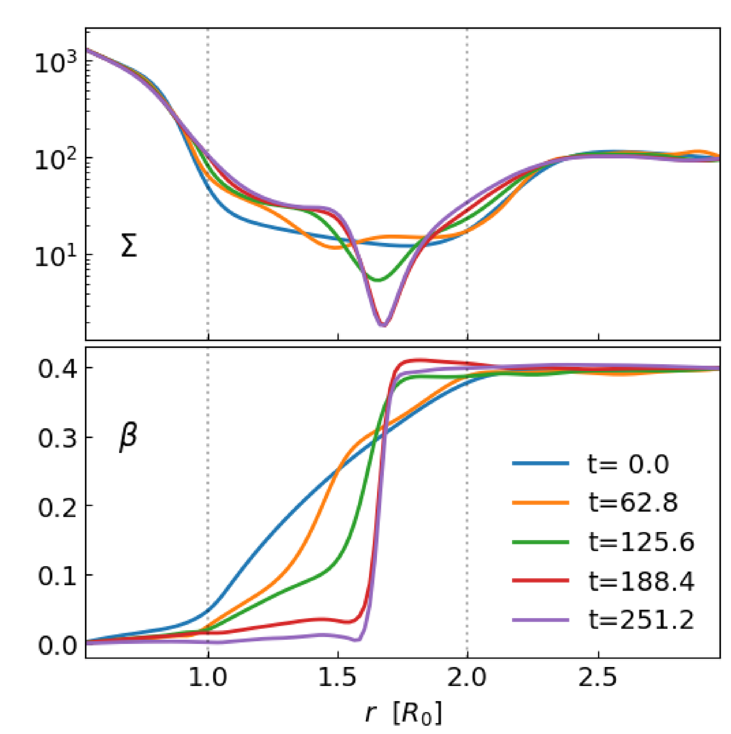


Figure 3. Simulation showing disk breaking (with $\beta_{out} = 0.4 = 23^o$).

- **Top left:** density cross section at $\theta = \pi/2$
- **Top right:** density cross section at $\phi = \pi/4$
- **Bottom:** time evolution of the radial profiles

A deep density gap and a sharp inclination jump emerges at r=1.5

Nonlinear Saturation and Bouncing: Route to Breaking

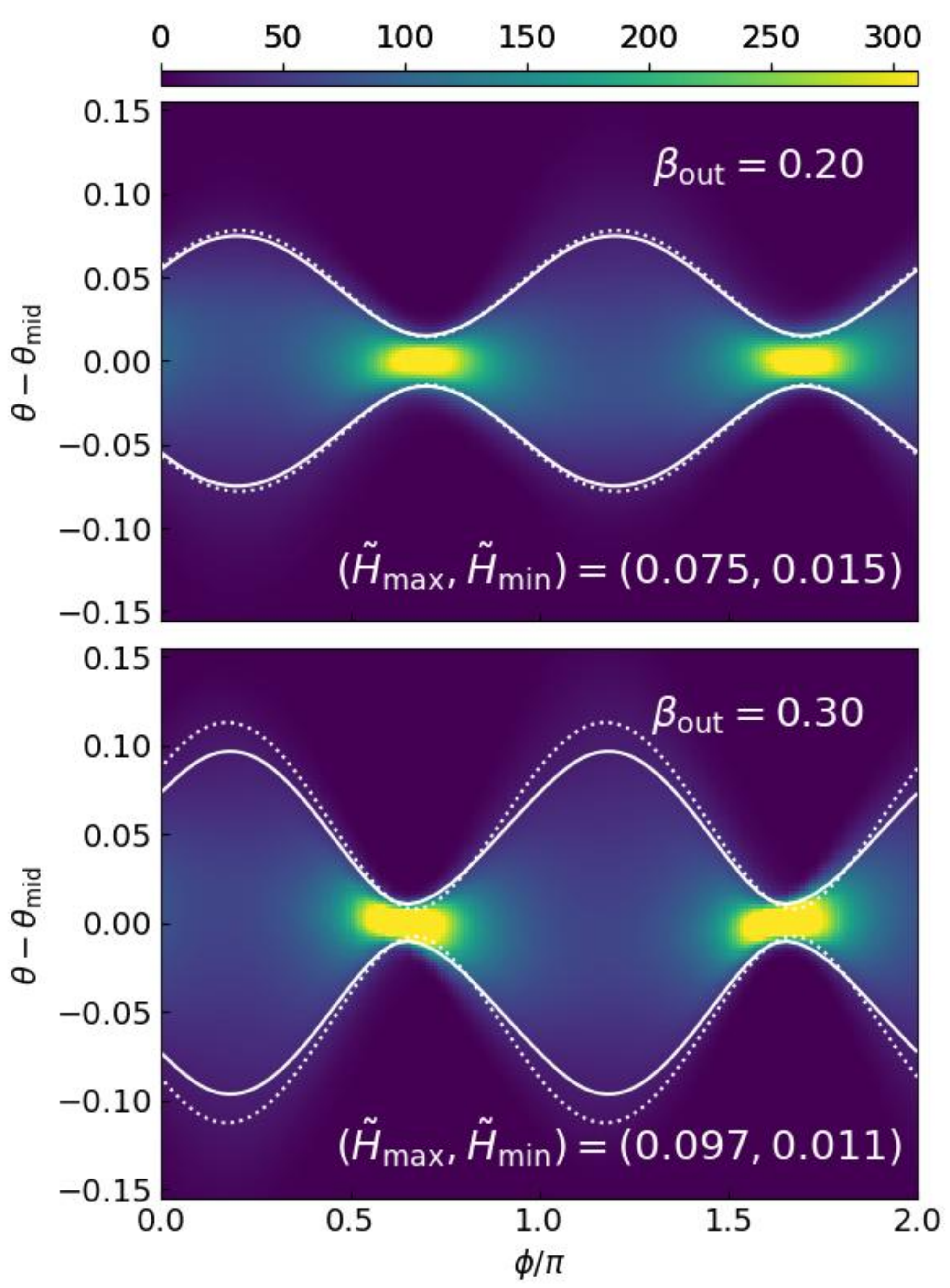


Figure 4 (right). Relation between the *internal torque* $|Q_4\psi|$ and *the warp amplitude* $|\psi|$.

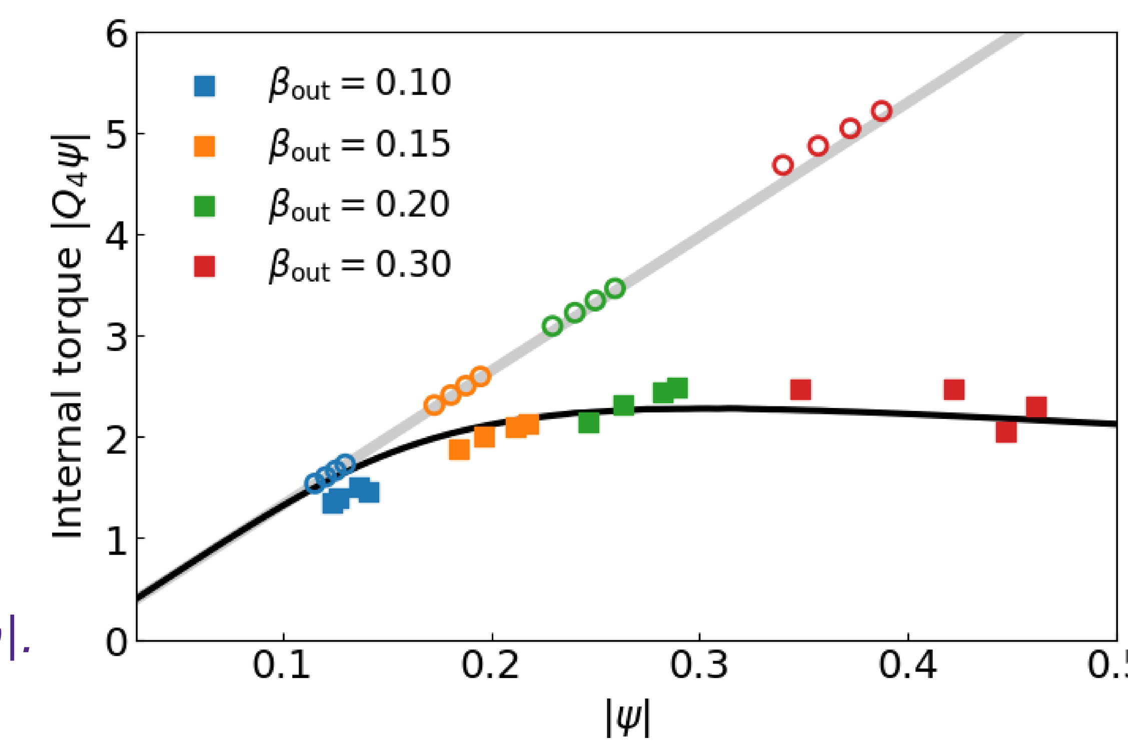
- **Scattered:** simulation at t=0 (circles) and t=125.6 (squares)
- **Curves:** Linear (gray) and nonlinear theories (black, based on Ogilvie & Latter 2013)

Torque saturates and its relation to $|\psi|$ becomes not one-to-one at large $|\psi|$.

Figure 5 (left). Density field at $r = 1.5$

- **Solid white:** measured scale height
- **Dashed white:** prediction based on Ogilvie & Latter (2013).

Gas undergoes extreme vertical compression and expansion, creating weak points prone to breaking.



Reference

Lubow & Ogilvie 2000, ApJ, **538**, 326
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